inkMaths

ANSWER BOOK

Sequences and sigma notation

Sequences and series · Year 12–13

7 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The terms are 1, 7, 17. For 71: $2n^2 - 1 = 71$ gives $n^2 = 36$, so $n = 6$, a positive whole number: yes, 71 is the sixth term. B1 for the three terms, M1 for setting $2n^2 - 1 = 71$ and A1 for $n = 6$ with the conclusion. Had n come out fractional, the answer would be no, and showing the failed equation would be the proof. [3]

A2 $u_2 = 2 \times 2 + 1 = 5$, then $u_3 = 2 \times 5 + 1 = 11$, then $u_4 = 2 \times 11 + 1 = 23$. M1 for a correct substitution into the recurrence, A1 for all three further terms. Show each feed of the machine. Recurrence questions are marked on the substitutions, so a bare list of four numbers can score less than working that contains one arithmetic slip. [2]

SECTION B · Standard

B1 The terms run 4, 3, 4, 3, so the sequence is periodic with period 2 and bounces between two values for ever. Dividing 12 by either value returns the other, so the pair traps itself. M1 for a correct division, A1 for the four terms, B1 for periodic with period 2. The word periodic and the number 2 are both wanted; describing it as “repeating” without the period is worth less. [3]

B2 The recipe gives 5, 8, 11, 14, 17, and the Σ says add them, so the total is 55. Splitting the sum works just as well: $3(1 + 2 + 3 + 4 + 5) + 2 \times 5 = 45 + 10 = 55$, since the constant 2 is added five times over. M1 for generating the terms or splitting the sum, A1 for the five values, A1 for 55. Treating $\Sigma 2$ as 2 rather than 10 is the standard error, and it is the reason the second route needs care. [3]

B3 $u_{12} = 50 - 48 = 2$. A negative term needs $50 - 4n < 0$, so $n > 12.5$, and since n counts terms the first negative one is the 13th, $u_{13} = -2$. The first five terms are 46, 42, 38, 34 and 30, which sum to 190. B1 for $u_{12} = 2$, M1 for $50 - 4n < 0$, A1 for the 13th term, M1 for the five terms, A1 for 190. Rounding 12.5 down to 12 is the trap: the inequality has to be satisfied, so n climbs to the next whole number. [5]

SECTION C · Stretch

C1 $u_{n+1} - u_n = 2(n + 1)/(n + 2) - 2n/(n + 1) = 2[(n + 1)^2 - n(n + 2)]/[(n + 1)(n + 2)] = 2/[(n + 1)(n + 2)]$, which is positive for every n: increasing. Also $u_n = 2 - 2/(n + 1) < 2$, since something positive is always subtracted. M1 for forming $u_{n+1} - u_n$, A1 for the simplified $2/[(n + 1)(n + 2)]$, A1 for stating it is positive, M1 for $u_n = 2 - 2/(n + 1)$, A1 for the conclusion. The sequence climbs for ever yet never reaches 2: increasing does not mean unbounded. [5]

C2 Split the sum: $2(1 + 2 + \ldots + 20) - (1 + 1 + \ldots + 1) = 2 \times 210 - 20 = 400$, using $\Sigma 1 = 20$ for the constant part. And $400 = 20^2$: the first n odd numbers always sum to n^2, each new odd number wrapping another layer round a square. M1 for splitting the sum, A1 for $2 \times 210 - 20$, A1 for 400, B1 for the link with n^2. [4]