



QUESTION WORKBOOK

Series solutions of differential equations

Further Pure 1 · Year FM

6 questions · 22 marks

PRINT EDITION

SECTION A · Warm-up

- A1** Describe the Taylor series method for solving a differential equation with given initial conditions. [2]

- A2** For $dy/dx = x + y^2$ with $y = 1$ at $x = 0$, find the values of the first and second derivatives of y at $x = 0$. [3]

SECTION B · Standard

- B1** For $dy/dx = x + y^2$ with $y = 1$ at $x = 0$, the first and second derivatives of y at $x = 0$ are 1 and 3. Find the third derivative at $x = 0$ and write the series solution up to the term in x^3 . [4]

B2 Use the substitution $z = y^2$ to solve $2y(dy/dx) + y^2 = x$. [4]

B3 For a second order equation, explain why two initial conditions are needed before the Taylor series method can begin. [2]

SECTION C · Stretch

C1 Given that $y'' + xy' + 2y = 0$ with $y = 1$ and $dy/dx = 0$ at $x = 0$, find the series solution for y up to and including the term in x^6 . Hence estimate the value of y at $x = 0.2$, giving your answer to five decimal places. [7]