inkMaths

ANSWER BOOK

Shortest paths: Dijkstra and Floyd

Decision Mathematics 1 · Year FM

7 questions · 34 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The order in which the vertex was given a permanent label, its final shortest distance from the start, and the working values tried along the way, each crossed out as a smaller one replaces it. B1 B1 B1 for the three parts of the label. The working values carry marks of their own, so they must be shown rather than tidied away. [3]

A2 Work back from F. 15 - 13 = 2, the weight of EF, so EF lies on the route. Then 13 - 7 = 6 = CE, and 7 - 0 = 7 = AC. The route is **A,** C, E, F of length **15**. M1 for testing label differences against the edge weights, A1 for the route, B1 for the length 15. Trace the route backwards, testing each edge against the difference of the labels at its ends, rather than reading it off the diagram by eye. [3]

SECTION B · Standard

B1 A is permanent first with 0, and its neighbours take working values B 5, C 7 and D 9. [8]
The smallest is B, permanent **2nd** at 5. From B, C would be 5 + 3 = 8, which does not beat 7, and E takes 5 + 9 = 14.
C is permanent **3rd** at 7. From C, D would be 7 + 3 = 10, no improvement on 9, and E improves from 14 to **13**.
D is permanent **4th** at 9. From D, E would be 9 + 5 = 14, no improvement, and F takes 9 + 10 = 19.
E is permanent **5th** at 13, and F improves from 19 to **15**.
F is permanent **6th** at 15. Working back, 15 - 13 = 2 = EF, 13 - 7 = 6 = CE and 7 = AC, so the shortest route is **A,** C, E, F of length **15**.
M1 for the first set of working values, A1 for B and C permanent at 5 and 7, M1 for improving E from 14 to 13, A1 for D permanent at 9, A1 for E permanent at 13, A1 for F permanent at 15, M1 for working the route back, A1 for A, C, E, F. Cross the 14 at E and the 19 at F out rather than erasing them. A correct final answer with a bare set of labels scores about half the marks.

B2 A, D, F has length 9 + 10 = **19**, against 15 for A, C, E, F. When D was made permanent with label 9, F received the working value 19. Later E was made permanent with 13, and 13 + 2 = 15 replaced it. B1 for the length 19, M1 for comparing it with 15, A1 for F taking the working value 19 from D, A1 for 13 + 2 = 15 replacing it. The 19 stays on the diagram crossed out, and it is the evidence that the algorithm was followed rather than the answer guessed. [4]

B3 It compares the current A to E entry with 7 + 6 = **13**. Any finite value beats no route at all, so the distance entry becomes **13** and the route entry for A to E is changed to **C**, the first vertex to head for. In an undirected network the matrices stay symmetric, so the entry from E to A changes in the same way. M1 for comparing with 7 + 6, A1 for the distance entry 13, A1 for the route entry C, B1 for the matching change from E to A. [4]

B4 All the weights are non-negative, so the smallest temporary label anywhere cannot be improved by any later route. Reaching that vertex another way would have to pass through a vertex carrying an equal or larger label first. Taking the cheapest edge from the current vertex instead is a greedy walk with no such guarantee, and one short edge into an expensive region can make it go badly wrong. B1 for the weights being non-negative, B1 for no later route improving the smallest label, B1 for the greedy alternative carrying no such guarantee. [3]

SECTION C · Stretch

C1 Initial distance matrix, rows and columns in the order A, B, C, D, with a dash where there is no edge. [9]
A: –, 3, 8, –
B: 3, –, 2, 5
C: 8, 2, –, 6
D: –, 5, 6, –
In the initial route matrix every entry is its own column label.
A: A, B, C, D
B: A, B, C, D
C: A, B, C, D
D: A, B, C, D
Iteration 1, through A. Compare each entry with the sum of its row's A entry and its column's A entry. B to C would be 3 + 8 = 11 against 2, and every route through A to D is blocked, so **nothing** changes. Copy both matrices out again.
Iteration 2, through B. A to C becomes 3 + 2 = **5**, beating 8, and A to D becomes 3 + 5 = **8**, beating no route at all. By symmetry C to A and D to A change with them. The distance matrix is now
A: –, 3, 5, 8
B: 3, –, 2, 5
C: 5, 2, –, 6
D: 8, 5, 6, –
and the route matrix becomes
A: A, B, **B**, **B**
B: A, B, C, D
C: **B**, B, C, D
D: **B**, B, C, D
B1 for the initial distance matrix, B1 for the initial route matrix, M1 for comparing each entry with the sum through A, A1 A1 for both matrices unchanged, M1 for the comparison through B, A1 for the new distances, A1 for the new route entries, A1 for both matrices written out in full. Write both matrices out after every iteration, including one that changes nothing, since a missing pair costs the marks for that iteration. The route entry records the first vertex on the way, not the last, so A to D reads B and the B row then sends you straight to D.