inkMaths

ANSWER BOOK

Statics of a particle

Mechanics · Year 12–13

6 questions · 27 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The resultant force on the particle is zero. In practice that means resolving in two perpendicular directions and setting each component sum to zero. B1 for the zero resultant, B1 for resolving in two perpendicular directions. Two directions give two equations, which is enough for two unknowns. [2]

A2 Resolving along the slope, $T = 30 \sin 25° = 12.7$ N. Perpendicular to the slope, $R = 30 \cos 25° = 27.2$ N. M1 for resolving along the slope, A1 for $T = 12.7$ N, M1 for resolving perpendicular to it, A1 for $R = 27.2$ N. Choosing the slope and its normal as the two directions keeps the unknowns apart, because each equation then contains only one of them. Resolving horizontally and vertically instead would give a pair of simultaneous equations for the same answer. [4]

SECTION B · Standard

B1 Horizontally, $T_1 \cos 20° = T_2 \cos 70°$. Vertically, $T_1 \sin 20° + T_2 \sin 70° = 10$. Solving the pair gives $T_1 = 3.42$ N and $T_2 = 9.40$ N. The steeper string carries far more of the load, which the sketch should have led you to expect. There is a neat check available here. The angles sum to 90°, so the strings are perpendicular, the weight is being split into two perpendicular components, and Pythagoras must reassemble it: $3.42^2 + 9.40^2 = 100 = 10^2$. Marks go for each resolving equation and for each tension, with the fifth for a correct method of solution. In codes that is M1 M1 for the two resolving equations, dM1 for solving them and A1 A1 for the two tensions. [5]

B2 The reaction is perpendicular to the slope, so it makes 30° with the vertical. Resolving horizontally, $R \sin 30° = P$. Resolving vertically, $R \cos 30° = 40$. Dividing the first by the second gives $P = 40 \tan 30° = 23.1$ N, and then $R = 40/\cos 30° = 46.2$ N. M1 for resolving horizontally, A1 for $R \sin 30° = P$, M1 for resolving vertically, A1 for $P = 23.1$ N, A1 for $R = 46.2$ N. The reaction exceeds the weight, because pressing horizontally into a slope squeezes the surface harder than gravity alone does. Any answer with R below 40 N has resolved the wrong way round. [5]

B3 Resolving perpendicular to the slope, $R = 3g \cos 35°$. Resolving along it, friction acts up the slope and $F = 3g \sin 35°$. Limiting equilibrium means $F = \mu R$, so $\mu = \tan 35° = 0.700$. M1 for resolving perpendicular to the slope, M1 for resolving along it, M1 for using $F = \mu R$ in limiting equilibrium, A1 for $\mu = 0.700$. The mass cancels, so any particle on the point of slipping on this slope gives the same answer: tilting a surface until an object just slides is a practical way of measuring μ, and the angle it slides at is independent of how heavy the object is. [4]

SECTION C · Stretch

C1 On the point of moving the particle is in limiting equilibrium, which is rest at the very moment of slipping. That is what adds the third equation $F = \mu R$ to the two resolving equations, and it is the only circumstance in which friction may be written at its maximum. Resolving vertically, the angled force lifts as well as pulls: $R + P \sin 20° = 4g$, so $R = 39.2 - P \sin 20°$. Resolving horizontally at the limit, $P \cos 20° = \mu R = 0.25(39.2 - P \sin 20°)$. Expanding, $P \cos 20° + 0.25P \sin 20° = 9.8$, so $P(0.9397 + 0.0855) = 9.8$ and $P = 9.8/1.0252 = 9.56$ N. The reaction is then 35.9 N, well below the weight of 39.2 N. B1 for stating limiting equilibrium with $F = \mu R$, M1 for resolving vertically, A1 for $R = 39.2 - P \sin 20°$, M1 for resolving horizontally at the limit, A1 for the equation in P, dM1 for solving it, A1 for $P = 9.56$ N. That reduction is the point of the question. Pulling at an angle lightens the load on the surface, lowers the friction ceiling, and so needs less force than the 9.8 N a purely horizontal pull would require. Writing $R = 4g$ and forgetting the $P \sin 20°$ term is the standard error, and it costs every mark after the first. [7]