InkMaths

ANSWER BOOK

Straight lines

Coordinate geometry · Year 12–13

7 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The gradient is m = (-3 - 3)/(4 - 1) = -2. Point-gradient form with (1, 3) gives y - 3 = -2(x - 1), which tidies to 2x + y - 5 = 0. Marks go to the gradient, the equation and the demanded integer form, so a correct y = -2x + 5 left unrearranged still loses one. M1 for the gradient, A1 for the equation, A1 for the integer form. Substituting the second point, 8 - 3 - 5 = 0, is a one-line check that catches most slips. [3]

A2 Perpendicular gradients multiply to -1, so each is the negative reciprocal. That gives -3/2 for the first and 1/4 for the second. B1 B1 for the two gradients. Flip the fraction, change the sign, and check the product really is -1. [2]

SECTION B · Standard

B1 The midpoint is (4, 6) and the segment's gradient is 8/6 = 4/3, so the bisector's gradient is -3/4. Through (4, 6): y - 6 = -(3/4)(x - 4), which clears to 3x + 4y - 36 = 0. B1 for the midpoint, M1 for the perpendicular gradient, M1 for the equation through (4, 6), A1 for 3x + 4y - 36 = 0. Both ingredients carry a mark. A bisector that uses one endpoint instead of the midpoint scores the gradient and nothing else. [4]

B2 Rearranging the given line, y = 2 - (2/3)x, so its gradient is -2/3 and the perpendicular gradient is 3/2. Through (4, -1): y + 1 = (3/2)(x - 4), which doubles to 2y + 2 = 3x - 12, giving 3x - 2y - 14 = 0. M1 for the gradient of the given line, A1 for the perpendicular gradient 3/2, A1 for 3x - 2y - 14 = 0. Reading the gradient straight off the coefficients as -a/b saves a line, provided the sign is quoted correctly. [3]

B3 The intercept 30 is the starting height in centimetres. The gradient -2.5 says the candle burns down 2.5 cm every hour. Setting H = 10 gives 2.5t = 20, so t = 8 hours. At t = 12 the model gives H = 0, and past that a negative height, so it cannot apply beyond 12 hours. B1 for the starting height, B1 for the rate with its direction, M1 for setting H = 10, A1 for t = 8, B1 for the model failing beyond 12 hours. Interpretation marks want units and a direction, so “gradient is -2.5” on its own is not enough. [5]

SECTION C · Stretch

C1 l_1 has gradient $-3/4$, so l_2 has gradient $4/3$ and, through the origin, equation $y = (4/3)x$. Substituting into $3x + 4y = 12$ gives $3x + (16/3)x = 12$, so $25x = 36$ and $x = 36/25$, $y = 48/25$. The meeting point is $(36/25, 48/25)$. Its distance from the origin is $\sqrt{(36^2 + 48^2)}/25 = 60/25 = 12/5 = 2.4$. M1 for the perpendicular gradient, A1 for $y = (4/3)x$, M1 for substituting, A1 for $(36/25, 48/25)$, M1 for the distance, A1 for $12/5$. The word shortest is what forces the perpendicular; a distance measured along any other line to l_1 is longer. The foot of the perpendicular has to be found first, and that extra stage is what makes this a six-mark question rather than a one-liner. [6]

C2 A vertical line has no gradient. The run is zero, so the fraction is undefined, and the rule compares two gradients that both have to exist. The lines are perpendicular by geometry, not by passing the product test. B1 for a vertical line having no gradient, B1 for the rule needing two gradients that exist. Writing $m = \infty$ and claiming $\infty \times 0 = -1$ earns nothing. [2]