inkMaths

QUESTION WORKBOOK

Subgroups, Lagrange's theorem and isomorphism

Further Pure 2 · Year FM

7 questions · 27 marks

PRINT EDITION

SECTION A · Warm-up

A1 State Lagrange's theorem and one immediate consequence for the order of an element. [2]

A2 A cyclic group of order 6 is generated by g. List its subgroups and check them against Lagrange's theorem. [3]

SECTION B · Standard

B1 A group G has order 8. State the possible orders of its subgroups, and prove that G cannot contain an element of order 3. [4]

B2 Prove that a group of prime order p is cyclic and has no proper subgroups other than the identity. [4]

B3 Decide whether the cyclic group of order 4 and the group $\{1, 3, 5, 7\}$ under multiplication modulo 8 are isomorphic, with a reason. [3]

B4 The symmetries of a square form a group of order 8. Find a subgroup of order 4 in two genuinely different ways, and one of order 2. [4]

SECTION C · Stretch

C1 The group D of the symmetries of a square has order 8. The set $U = \{1, 3, 5, 7, 9, 11, 13, 15\}$ forms a group of order 8 under multiplication modulo 16. Determine, with full justification, whether D and U are isomorphic. [7]