



ANSWER BOOK

Systems of equations and invariance

Matrices · Year FM

7 questions · 31 marks

NAVY EDITION

SECTION A · Warm-up

- A1** M has rows (2, 1) and (1, 3), v is the column (x, y) and b the column (7, 11). B1 B1 [3]
B1 for M, v and b. One matrix equation carries both scalar equations at once.
- A2** $\det M = 6 - 1 = 5$, so $M^{-1} = (1/5) \times \text{rows } (3, -1), (-1, 2)$. Then $v = M^{-1}b = (1/5)(21 - 11, -7 + 22) = (2, 3)$. Check: $2 \times 2 + 3 = 7$ and $2 + 9 = 11$. M1 for $\det M = 5$, A1 for the inverse, M1 for $v = M^{-1}b$, A1 for (2, 3). [4]

SECTION B · Standard

- B1** Either a prism, where the three pairwise intersection lines are parallel and the [4]
planes fence off a triangular tube, or two planes meeting in a line that the third misses
everywhere by being parallel to it. The prism is the configuration with three parallel
intersection lines; both give an inconsistent system with a singular coefficient matrix. B1
B1 for the two configurations, B1 for naming the prism, B1 for the singular coefficient
matrix.
- B2** Take (1, 1) on the line. It maps to (5, 5), still on $y = x$, and any multiple (t, t) maps [4]
to (5t, 5t) by linearity. The line maps into itself with stretch factor 5. M1 for mapping a
general point of the line, A1 for the image (5t, 5t), A1 for the conclusion, B1 for the stretch
factor 5. This is an invariant line rather than a line of fixed points. Every point on it
moves, but none of them leaves the line.
- B3** Try $y = mx$. The image of (1, m) is (4 + m, 2 + 3m), which lies on $y = mx$ when 2 + [4]
3m = m(4 + m), so $m^2 + m - 2 = 0$, giving $m = 1$ or $m = -2$. The new line is $y = -2x$,
stretched by factor 2, since (1, -2) maps to (2, -4). M1 for trying $y = mx$, A1 for $m^2 + m - 2$
= 0, A1 for $y = -2x$, B1 for the stretch factor 2.
- B4** On an invariant line each point maps to a point of the same line, possibly [4]
elsewhere; a line of invariant points maps every point to itself. For a reflection, the mirror
line is a line of invariant points, while any line perpendicular to the mirror is invariant
without its points being fixed. B1 B1 for the two definitions, B1 B1 for the mirror line and a
perpendicular line as the two examples.

SECTION C · Stretch

- C1** Expanding along the top row, $\det M = (2k - 9) - (k - 6) + (3 - 4) = k - 4$, so M is [8]
singular when $k = 4$. For any other k the inverse exists and there is a single point of
intersection. With $k = 4$ the third equation reads $2x + 3y + 4z = 5$, which is exactly the
sum of the first two, so it carries no new information and the system is **consistent**.
Solving the first two, subtracting gives $y + 2z = 3$, so $y = 3 - 2z$ and then $x = 1 - y - z = -2$
 $+ z$. Writing $z = \lambda$, the solutions form the line $r = (-2, 3, 0) + \lambda(1, -2, 1)$. Geometrically the
three planes form a sheaf, meeting in that common line. M1 for expanding the
determinant, A1 for $\det M = k - 4$, A1 for $k = 4$, M1 for testing consistency, A1 for the third
equation being the sum of the first two, M1 for solving the first two, A1 for the line of
solutions, B1 for the sheaf. A singular matrix rules out a unique solution but says nothing
about consistency; the sheaf here and an inconsistent prism both give $\det M = 0$, so the
equations still have to be tested.