inkMaths

ANSWER BOOK

Testing a correlation coefficient

Further Statistics 2 · Year FM

7 questions · 29 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $H_0: \rho = 0$ and $H_1: \rho > 0$, where ρ is the population correlation coefficient. The sample value r is what was measured, so it is the evidence rather than the claim. B1 B1 for the two hypotheses, B1 for ρ being the population coefficient. Putting r in a hypothesis asserts something already known, so it makes no testable claim. [3]

A2 $H_0: \rho = 0$; $H_1: \rho > 0$, one-tailed at 5% with $n = 12$. Since $0.62 > 0.4973$ the result lies in the critical region, so **reject** H_0: there is evidence at the 5% level of positive correlation between the two variables in the population. B1 for the hypotheses, M1 for comparing 0.62 with 0.4973, A1 for rejecting H_0 with a conclusion in context. [3]

SECTION B · Standard

B1 $H_0: \rho = 0$; $H_1: \rho \neq 0$, two-tailed at 5%, so 2.5% goes in each tail and the critical values are ± 0.7067. Since $0.65 < 0.7067$, the result is not in the critical region, so **do** not reject H_0. There is insufficient evidence at the 5% level of correlation between the two variables in the population. B1 for the hypotheses, B1 for splitting the 5% into two tails, M1 for comparing 0.65 with 0.7067, A1 for not rejecting H_0 in context. A one-tailed test at the same level would have used 0.6215 and rejected, so settle the tail from the wording of the question before looking anything up. [4]

B2 $H_0: \rho_s = 0$; $H_1: \rho_s > 0$, one-tailed at 5% with $n = 10$. Since $0.62 > 0.5636$, **reject** H_0: there is evidence at the 5% level that the rankings agree. Because the test uses only the orders, it makes no assumption about the shape of the underlying distributions and is unaffected by a single extreme value. B1 for the hypotheses, M1 for comparing 0.62 with 0.5636, A1 for rejecting H_0 in context, B1 for the distribution-free advantage. [4]

B3 The critical values assume the pairs are drawn from a population with a bivariate normal distribution. Formal verification is not expected, but the assumption should be stated as part of the conclusion, since the test is not valid without it. Spearman's test requires no such assumption. B1 for bivariate normality, B1 for stating it in the conclusion, B1 for Spearman's test needing no such assumption. [3]

B4 With $n = 10$ the value 0.35 falls short of 0.5494, so H_0 is not rejected. With $n = 30$ the same value exceeds 0.3061, so H_0 is rejected. The identical coefficient is significant in one case and not in the other, so a coefficient cannot be interpreted without its sample size. A small sample quite easily produces a moderate correlation by chance; a large one still can, though far less often, which is why the critical value falls with n rather than vanishing. Significance and strength are separate questions. M1 for comparing 0.35 with both critical values, A1 for the two opposite decisions, B1 for the sample size mattering, B1 for separating significance from strength. [4]

SECTION C · Stretch

C1 Differences d: −2, 1, 1, −1, 1, −1, 1, 0. Squaring gives 4, 1, 1, 1, 1, 1, 1, 0, so $\Sigma d^2 =$ **10**. [8]
$r_s = 1 - 6\Sigma d^2/[n(n^2 - 1)] = 1 - 60/(8 \times 63) = 1 - 60/504 =$ **0.881** to three decimal places.
H_0: $\rho_s = 0$; H_1: $\rho_s > 0$, one-tailed at 5% with $n = 8$, since the question asks about agreement rather than any association.
Since $0.881 > 0.6429$ the result is in the critical region, so **reject** H_0. There is evidence at the 5% level that the judges' rankings agree.
M1 for the differences, A1 for $\Sigma d^2 = 10$, M1 for the formula, A1 for 0.881, B1 B1 for the two hypotheses, M1 for comparing with 0.6429, A1 for rejecting H_0 in context. Set the differences out in a table and show Σd^2 before substituting. Signs do not matter once they are squared, but a single arithmetic slip in Σd^2 carries through to everything after it, so total the d column and check it comes to zero.