inkMaths

ANSWER BOOK

The binomial expansion

Sequences and series · Year 12–13

7 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $8!/(3! \times 5!) = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 336/6 = 56$. M1 for the cancelled quotient, A1 for 56. The 5! wipes out all but three factors of the top, so only $8 \times 7 \times 6$ survives. Computing $8! = 40\,320$ in full is the long way round and invites arithmetic slips. [2]

A2 1, 5, 10, 10, 5, 1, which is six entries for six terms. B1 for the row. Each entry is the sum of the two above it, and the row reads the same in both directions. A power of 5 needs the row starting 1, 5, so counting the apex as row 1 and reaching 1, 4, 6, 4, 1 is the off-by-one to avoid. [1]

SECTION B · Standard

B1 Row 4 gives 1, 4, 6, 4, 1. Build each term with the powers of 3 falling and the powers of 2x climbing: 81, then $4 \times 27 \times 2x = 216x$, then $6 \times 9 \times 4x^2 = 216x^2$, then $4 \times 3 \times 8x^3 = 96x^3$, then $16x^4$. So $(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$. Setting $x = 1$ turns the bracket into $5^4 = 625$, and $81 + 216 + 216 + 96 + 16 = 625$, so the check passes for the price of one line. B1 for the row of coefficients, M1 for a term with both powers correct, A1 for the middle terms, A1 for the full expansion. Every term needs the 2 raised as well as the x; leaving 2x as 2 in the later terms is the usual collapse. [5]

B2 $1 + {}^8C_1(3x) + {}^8C_2(3x)^2 = 1 + 24x + 28 \times 9x^2 = 1 + 24x + 252x^2$. M1 for the binomial coefficients with 3x substituted whole, A1 for 24x, A1 for $252x^2$. The replacement 3x goes into the formula whole and gets squared whole. Forgetting to square the 3 gives $84x^2$ and costs the final mark, which is the single commonest loss in this topic. [3]

B3 The terms are $2^5 = 32$, then $5 \times 2^4x = 80x$, then $10 \times 2^3x^2 = 80x^2$. Setting $x = 0.002$: $32 + 0.16 + 0.00032 = 32.16032$. The true value is 32.1603203…, so the truncation is already good to six decimal places; the discarded x^3 term is only 40×0.002^3, three parts in ten million. M1 for a term with the falling power of 2, A1 for 80x, A1 for $80x^2$, M1 for substituting $x = 0.002$, A1 for 32.16032. [5]

SECTION C · Stretch

C1 The x^2 term is ${}^6C_2 \times 2^4 \times (kx)^2 = 15 \times 16 \times k^2x^2 = 240k^2x^2$. Setting $240k^2 = 60$ gives $k^2 = ¼$, so $k = ½$ or $k = -½$. Both are wanted, since only k^2 appears and the sign cannot be recovered from an even power. At $k = ½$ the expansion opens $64 + 96x + 60x^2$, which confirms the coefficient. M1 for the x^2 term of the expansion, A1 for $240k^2$, M1 for solving $240k^2 = 60$, A1 for both values of k. The mark at risk is the 2^4: the power on the first term drops as the power on the second climbs, and answers that leave it at 2^6 or omit it entirely reach a different k. [4]

C2 The coefficient ${}^nC_r$ counts the ways of choosing which r brackets supply the second letter; choosing those r is the same act as choosing the other n - r to supply the first. So ${}^nC_r = {}^nC_{n-r}$, and the row is a palindrome. B1 for choosing r brackets being the same act as choosing the other n - r, B1 for the resulting symmetry of the row. [2]