inkMaths

ANSWER BOOK

The general binomial expansion

Sequences and series · Year 12–13

6 questions · 26 marks

NAVY EDITION

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SECTION A · Warm-up

A1 For whole n the coefficient $n(n - 1)\ldots(n - r + 1)/r!$ eventually picks up the factor zero, and every term after that dies. For $n = -1$ the factors run $-1, -2, -3, \ldots$ and never hit zero, so the series continues for ever. It then only converges for $|x| < 1$, and quoting that condition is a mark in its own right on every question of this type. B1 B1 for the two explanations, B1 for $|x| < 1$. [3]

A2 With $n = -1$ the expansion is $1 - x + x^2 - x^3 + \ldots$, valid for $|x| < 1$. M1 for the expansion, A1 for $1 - x + x^2 - x^3$, B1 for the validity. The alternating signs come from the coefficient factors $-1, -2, -3$ dividing by $1!$, $2!$ and $3!$, each ratio working out at ±1. This is the geometric series for $1/(1 + x)$ in binomial clothing, first term 1 and common ratio $-x$, and the same $|r| < 1$ condition appears from both directions. [3]

SECTION B · Standard

B1 Apply the formula with $n = ½$ and $4x$ in place of x: $1 + ½(4x) + [½ \times (-½)/2](4x)^2 = 1 + 2x - 2x^2$. Validity needs $|4x| < 1$, so $|x| < ¼$. M1 for the formula with $n = ½$, A1 for the x term, A1 for the x^2 term, B1 for the validity. The replacement goes in whole and gets squared whole, and the validity window shrinks by the same factor of 4. Answers that square only the x reach $-x^2/8$ and lose two marks together. [4]

B2 With $n = -2$ and $2x$ in place of x: $1 + (-2)(2x) + [(-2)(-3)/2](2x)^2 = 1 - 4x + 12x^2$, valid for $|x| < ½$. Setting $x = 0.01$: $1 - 0.04 + 0.0012 = 0.9612$, against a true value of $0.961169\ldots$: agreement to four decimal places from three terms. M1 for the expansion, A1 for $1 - 4x + 12x^2$, B1 for the validity, M1 for setting $x = 0.01$, A1 for 0.9612. [5]

B3 The bracket does not start with 1, so factor the 4 out: $(4 + x)^{1/2} = 2(1 + x/4)^{1/2}$. Expanding the inner bracket: $1 + x/8 - x^2/128 + \ldots$, and doubling gives $2 + x/4 - x^2/64$. Validity needs $|x/4| < 1$, so $|x| < 4$. M1 for taking the 4 outside, A1 for the factor 2, M1 for expanding the inner bracket, A1 for $2 + x/4 - x^2/64$, B1 for the validity. Forgetting that the factored-out 4 comes through as $4^{1/2} = 2$, not 4, is where this question usually goes wrong. As a sanity check, $x = 0.2$ gives 2.049375 against a true $\sqrt{4.2} = 2.0493901\ldots$, so three terms are already worth four decimal places. [5]

SECTION C · Stretch

C1 Treat the denominator as a negative power: the expression is $(1 + 2x)(1 - 4x)^{-1/2}$. Expanding the second factor with $n = -\frac{1}{2}$ and $-4x$ in place of x gives $1 + (-\frac{1}{2})(-4x) + [(-\frac{1}{2})(-3/2)/2](-4x)^2 = 1 + 2x + 6x^2$. Now multiply by $(1 + 2x)$, keeping only terms as far as x^2: the x term is $2x + 2x = 4x$, and the x^2 term is $6x^2 + 4x^2 = 10x^2$. So the expansion is $1 + 4x + 10x^2$. M1 for writing the denominator as a negative power, M1 for expanding it, A1 for $1 + 2x + 6x^2$, M1 for multiplying out as far as x^2, A1 for the x term, A1 for the x^2 term. Two traps sit here. The minus sign inside the bracket has to survive the squaring, since $(-4x)^2$ is $+16x^2$; and the multiplying-out stage must not drop the cross term $2x \times 2x$. The validity $|4x| < 1$ is exactly the $|x| < \frac{1}{4}$ the question hands you, which is a hint that the $-4x$ has been read correctly. [6]