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QUESTION WORKBOOK

# The Newton–Raphson method

Numerical methods · Year 13

15 questions · 57 marks

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## SECTION A · Warm-up

**A1** NRT-A1 Write down the Newton-Raphson iteration formula for solving $f(x) = 0$. [2]

**A2** NRT-A2 The tangent to the curve $y = f(x)$ at the point $(x_n, f(x_n))$ has equation $y - f(x_n) = f'(x_n)(x - x_n)$. Show that this tangent meets the x-axis at $x = x_n - f(x_n)/f'(x_n)$. [2]

**A3** NRT-A3 $f(x) = x^2 - 5$. Taking $x_0 = 2$, apply the Newton-Raphson method once, and give $x_1$ as an exact fraction. [2]

**A4** NRT-A4 State what one step of the Newton-Raphson method does geometrically, and state what happens to that step when $f'(x_n)$ is small but not zero. [3]

## SECTION B · Standard

**B1** NRT-B1

$f(x) = x^3 + x - 5$. Using the Newton-Raphson method with $x_0 = 1.5$, find $x_1$ and $x_2$, giving $x_2$ to 5 decimal places. [4]

**B2** NRT-B2

$f(x) = \ln x + x - 3$. Show that $f(x) = 0$ has a root in the interval $(2, 3)$, and use the Newton-Raphson method with $x_0 = 2$ to find $x_1$ and $x_2$, giving $x_2$ to 4 decimal places. [4]

**B3** NRT-B3

$f(x) = x^3 - 2x + 2$. Find the iterates $x_1$ and $x_2$ given by the Newton-Raphson method from $x_0 = 0$, and state what they show about the method from this starting value. [2]

**B4** NRT-B4

$f(x) = e^x - 3x$. Given that $f(x) = 0$ has a root between $x = 1.5$ and $x = 2$, use the Newton-Raphson method with $x_0 = 1.5$ to find $x_1$ and $x_2$, giving $x_2$ to 5 decimal places. [4]

**B5** NRT-B5 $f(x) = x^3 - 6x^2 + 9x - 1$. Explain why no Newton-Raphson iterate can be found from $x_0 = 3$, and find $x_1$ from $x_0 = 3.2$, giving your answer to 4 decimal places. [4]

**B6** NRT-B6 A parachutist falls from rest, and the distance fallen after t seconds is modelled by $s = 50t + 250(e^{-0.2t} - 1)$ metres. Given that the time taken to fall 100 m lies between 5 and 6 seconds, use the Newton-Raphson method with $t_0 = 5$ to find that time to 3 significant figures. [4]

**B7** NRT-B7 $f(x) = x^3 - 2x - 5$, and the root of $f(x) = 0$ is $\alpha = 2.094551$ correct to 6 decimal places. Using the Newton-Raphson method with $x_0 = 2$, find $x_1$ and $x_2$, giving $x_2$ to 6 decimal places, and compare how close $x_0$, $x_1$ and $x_2$ each lie to $\alpha$. [5]

## SECTION C · Stretch

**C1** NRT-C1

$f(x) = x^3 - 3x + 1$. Explain why the Newton-Raphson method cannot be applied with $x_0 = 1$. Taking $x_0 = 0.9$ instead, find $x_1$ and comment on the result, given that $f(x) = 0$ has roots near -1.879, 0.347 and 1.532. [6]

**C2** NRT-C2

$f(x) = \ln x$, so that $f(x) = 0$ has the single root $x = 1$. Show that the Newton-Raphson method gives $x_{n+1} = x_n(1 - \ln x_n)$, and explain why the method breaks down when it is started at $x_0 = 3$. [3]

**C3** NRT-C3

Show that applying the Newton-Raphson method to $f(x) = x^2 - a$, for a constant $a > 0$, gives the iteration $x_{n+1} = \frac{1}{2}(x_n + a/x_n)$. Taking $a = 10$ and $x_0 = 3$, find $x_1$, $x_2$ and $x_3$, giving $x_3$ to 6 decimal places, and show that $x_{n+1} - \sqrt{a} = (x_n - \sqrt{a})^2/(2x_n)$. [6]

**C4**
NRT-C4

A spherical tank of radius 2 m holds liquid to a depth of h m, and the volume held is $V = \pi h^2(6 - h)/3$ cubic metres for $0 \le h \le 4$. Find the depth at which the tank holds 10 $m^3$, to 3 significant figures, using the Newton-Raphson method from $h_0 = 1.5$, and explain why the equation has two further roots the model cannot use. [6]

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