



QUESTION WORKBOOK

The Newton–Raphson method

Numerical methods · Year 13

15 questions · 57 marks

NAVY SCREEN EDITION

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SECTION A · Warm-up**A1**

NRT-A1

Write down the Newton-Raphson iteration formula for solving $f(x) = 0$.

[2]

A2

NRT-A2

The tangent to the curve $y = f(x)$ at the point $(x_n, f(x_n))$ has equation $y - f(x_n) = f'(x_n)(x - x_n)$. Show that this tangent meets the x -axis at $x = x_n - f(x_n)/f'(x_n)$.

[2]

A3

NRT-A3

 $f(x) = x^2 - 5$. Taking $x_0 = 2$, apply the Newton-Raphson method once, and give x_1 as an exact fraction.

[2]

A4

NRT-A4

State what one step of the Newton-Raphson method does geometrically, and state what happens to that step when $f'(x_n)$ is small but not zero.

[3]

SECTION B · Standard

- B1** $f(x) = x^3 + x - 5$. Using the Newton-Raphson method with $x_0 = 1.5$, find x_1 and x_2 , giving x_2 to 5 decimal places. [4]
NRT-B1

- B2** $f(x) = \ln x + x - 3$. Show that $f(x) = 0$ has a root in the interval $(2, 3)$, and use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2 , giving x_2 to 4 decimal places. [4]
NRT-B2

- B3** $f(x) = x^3 - 2x + 2$. Find the iterates x_1 and x_2 given by the Newton-Raphson method from $x_0 = 0$, and state what they show about the method from this starting value. [2]
NRT-B3

- B4** $f(x) = e^x - 3x$. Given that $f(x) = 0$ has a root between $x = 1.5$ and $x = 2$, use the Newton-Raphson method with $x_0 = 1.5$ to find x_1 and x_2 , giving x_2 to 5 decimal places. [4]
NRT-B4

- B5** $f(x) = x^3 - 6x^2 + 9x - 1$. Explain why no Newton-Raphson iterate can be found from $x_0 = 3$, and find x_1 from $x_0 = 3.2$, giving your answer to 4 decimal places. [4]
NRT-B5

- B6** A parachutist falls from rest, and the distance fallen after t seconds is modelled by $s = 50t + 250(e^{-0.2t} - 1)$ metres. Given that the time taken to fall 100 m lies between 5 and 6 seconds, use the Newton-Raphson method with $t_0 = 5$ to find that time to 3 significant figures. [4]
NRT-B6

- B7** $f(x) = x^3 - 2x - 5$, and the root of $f(x) = 0$ is $\alpha = 2.094551$ correct to 6 decimal places. Using the Newton-Raphson method with $x_0 = 2$, find x_1 and x_2 , giving x_2 to 6 decimal places, and compare how close x_0 , x_1 and x_2 each lie to α . [5]
NRT-B7

SECTION C · Stretch

C1
NRT-C1

$f(x) = x^3 - 3x + 1$. Explain why the Newton–Raphson method cannot be applied with $x_0 = 1$. Taking $x_0 = 0.9$ instead, find x_1 and comment on the result, given that $f(x) = 0$ has roots near -1.879 , 0.347 and 1.532 . [6]

C2
NRT-C2

$f(x) = \ln x$, so that $f(x) = 0$ has the single root $x = 1$. Show that the Newton–Raphson method gives $x_{n+1} = x_n(1 - \ln x_n)$, and explain why the method breaks down when it is started at $x_0 = 3$. [3]

C3
NRT-C3

Show that applying the Newton–Raphson method to $f(x) = x^2 - a$, for a constant $a > 0$, gives the iteration $x_{n+1} = \frac{1}{2}(x_n + a/x_n)$. Taking $a = 10$ and $x_0 = 3$, find x_1 , x_2 and x_3 , giving x_3 to 6 decimal places, and show that $x_{n+1} - \sqrt{a} = (x_n - \sqrt{a})^2 / (2x_n)$. [6]

C4

NRT-C4

A spherical tank of radius 2 m holds liquid to a depth of h m, and the volume held is $V = \pi h^2(6 - h)/3$ cubic metres for $0 \leq h \leq 4$. Find the depth at which the tank holds 10 m^3 , to 3 significant figures, using the Newton-Raphson method from $h_0 = 1.5$, and explain why the equation has two further roots the model cannot use. [6]

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