inkMaths

ANSWER BOOK

The normal distribution

Statistics · Year 12–13

6 questions · 21 marks

NAVY EDITION

inkphysics.co.uk/maths/ · youtube.com/@InkPhysics

SECTION A · Warm-up

A1 A single symmetric peak at the mean, falling away identically on both sides, with the mean, median and mode all at the same place. About 68% of values lie within one standard deviation of the mean and about 95% within two. B1 for the symmetry about the mean, B1 for 68%, B1 for 95%. These two figures are worth memorising, because they catch a calculator answer that is wildly out before it costs anything. [3]

A2 Standardise: $z = (70 - 60)/5 = 2$, so $P(X < 70) = P(Z < 2) = 0.977$. M1 for standardising, A1 for 0.977. Two standard deviations above the mean leaves only about 2.3% in the upper tail, so an answer just below 1 is what the sketch predicts. Write the standardising line out even when the calculator will take μ and σ directly, because the method mark is attached to it. [2]

SECTION B · Standard

B1 Standardise both ends: $z = (65 - 60)/10 = 0.5$ and $z = (75 - 60)/10 = 1.5$. Then $P(0.5 < Z < 1.5) = P(Z < 1.5) - P(Z < 0.5) = 0.9332 - 0.6915 = 0.242$. M1 for standardising both ends, M1 for the subtraction, A1 for 0.242. Subtract the smaller cumulative probability from the larger; adding them, or subtracting the z values, are both worth nothing. Shade the strip on a sketch first and the subtraction writes itself. [3]

B2 The top 5% of the standard normal starts at $z = 1.6449$. So $(46 - 40)/\sigma = 1.6449$, giving $\sigma = 6/1.6449 = 3.65$. B1 for $z = 1.6449$, M1 for the standardising equation, M1 for rearranging, A1 for 3.65. Check forwards: 46 is then 1.64 standard deviations above the mean, which cuts off 5%, as stated. Quote the z value before forming the equation, and note that z here is positive because 46 lies above the mean. [4]

B3 A probability of 0.975 below 50 puts 50 at $z = +1.96$, so $(50 - \mu)/4 = 1.96$ and $\mu = 50 - 7.84 = 42.16$. M1 for the standardising equation, M1 for rearranging, A1 for 42.16. The value 50 sits above the mean, not below it. Reading 0.975 as a lower-tail area is what keeps the sign of z honest, and a negative z here would give $\mu = 57.84$, comfortably above a value that 97.5% of the distribution falls below. [3]

SECTION C · Stretch

C1 Turn each probability into a z value first. The bottom 10% ends at $z = -1.2816$, so $(20 - \mu)/\sigma = -1.2816$. The top 5% begins at $z = 1.6449$, so $(35 - \mu)/\sigma = 1.6449$. Rewrite them as $20 = \mu - 1.2816\sigma$ and $35 = \mu + 1.6449\sigma$. Subtracting removes μ: $15 = 2.9265\sigma$, so $\sigma = 5.13$. Then $\mu = 20 + 1.2816 \times 5.1257 = 26.6$. Check both statements before finishing: $(35 - 26.57)/5.126 = 1.645$, as required. B1 B1 for the two z values, M1 for the pair of simultaneous equations, M1 for the elimination, A1 for σ, A1 for μ. Sign is where this question is won and lost. The 20 is below the mean, so its z must be negative. Writing $+1.2816$ there gives $\sigma = 41.3$ and a negative mean, which is plainly impossible for a variable whose values sit around 20 to 35. [6]