inkMaths

ANSWER BOOK

The Poisson distribution

Further Statistics 1 · Year FM

6 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Events must occur singly, at random, independently of one another, and at a constant average rate over the window in question. The only parameter is that mean rate λ, which is also the variance. B1 B1 for the four conditions, B1 for naming λ. All four conditions are needed, and 'independently' and 'at a constant rate' are the two most often left out. [3]

A2 $P(X = 3) = e^{-4} \times 4^3/3! = 0.1954$. $P(X \leq 2) = e^{-4}(1 + 4 + 8) = 0.2381$. M1 for the Poisson formula, A1 for 0.1954, A1 for 0.2381. The cumulative value comes fastest from a calculator's own Poisson function. [3]

SECTION B · Standard

B1 Scale λ with the window. Over two minutes the count is $Po(2.5 \times 2)$ = **$Po(5)$**, so $P(X = 0) = e^{-5}$ = **0.0067** and the variance is **5**, equal to the mean. M1 for scaling λ, A1 for $Po(5)$, A1 for 0.0067, B1 for the variance. Rescaling λ to the interval actually asked about is the step that decides the whole answer, and leaving it at 2.5 gives 0.0821. [4]

B2 $P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.7440 = 0.2560$. M1 for the one minus form, A1 for 0.2560. Converting an 'at least' to one minus an 'at most', with the boundary dropped by one, is the standard manoeuvre. [2]

B3 n is large and p small, so $B(200, 0.01) \approx Po(2)$ with $\lambda = np = 2$. Then $P(X \leq 1) = e^{-2}(1 + 2) = 0.4060$. The exact binomial gives 0.4046, so the approximation is out by 0.0014: close enough for any practical purpose. B1 for $\lambda = 2$, M1 for the Poisson probability, A1 for 0.4060, B1 for the binomial value, A1 for the comparison. [5]

SECTION C · Stretch

C1 Independent Poisson variables add, so the two sources pool to Po(1.5 + 2) = Po(3.5) per hour. Scaling the window to three hours multiplies the rate by 3, giving **Po(10.5)** for the shift. [7]

Both the mean and the variance are **10.5**. A Poisson distribution has only one parameter, and it serves as both, so a sample whose variance is far from its mean is evidence against the model.

$P(X \le 8)$ = **0.279** to three decimal places, from the cumulative Poisson function with $\lambda = 10.5$.

M1 for pooling the rates, M1 for scaling to three hours, A1 for Po(10.5), B1 for the mean and variance, B1 for why they are equal, M1 for the cumulative probability, A1 for 0.279.

Two steps have to happen in the right order. Pool the rates first and then scale, or scale each and then pool, but never scale only one of them. The additive property needs the two sources to be independent, and that should be stated.