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ANSWER BOOK

The t-formulae

Further Pure 1 · Year FM

6 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $\sin\theta = 2t/(1 + t^2)$, $\cos\theta = (1 - t^2)/(1 + t^2)$ and $\tan\theta = 2t/(1 - t^2)$. B1 B1 B1 for the three formulae. All three share the same right triangle, with legs $2t$ and $1 - t^2$ and hypotenuse $1 + t^2$. [3]

A2 $1 + t^2 = 5$, so $\sin\theta = 4/5$ and $\cos\theta = (1 - 4)/5 = -3/5$. M1 for $1 + t^2 = 5$, A1 for $\sin\theta$, A1 for $\cos\theta$. A negative cosine with a positive sine puts θ in the second quadrant, which a sketch confirms since $t > 1$ means $\theta/2$ exceeds 45°. [3]

SECTION B · Standard

B1 Substituting and multiplying by $1 + t^2$: $3(1 - t^2) - 8t = 5(1 + t^2)$, so $8t^2 + 8t + 2 = 0$, that is $(2t + 1)^2 = 0$. The repeated root $t = -1/2$ gives $\theta = 2\arctan(-1/2) = -0.927$, which is 5.36 radians in the required interval. M1 for substituting the t-formulae, M1 for clearing the denominator, A1 for $8t^2 + 8t + 2 = 0$, M1 for solving, A1 for $t = -1/2$, A1 for 5.36. A repeated root was inevitable here. Since $\sqrt{(3^2 + 4^2)} = 5$ is the greatest value the left side can take, the line is reached tangentially rather than crossed, so the two solutions coincide. [6]

B2 $\tan(\pi/2)$ is undefined, so $\theta = \pi$ corresponds to no finite value of t, so the substitution cannot reach it at all. Whenever the interval contains π, substitute $\theta = \pi$ into the original equation by hand and check separately whether it is a solution. B1 for $\tan(\pi/2)$ being undefined, B1 for no finite t reaching $\theta = \pi$, B1 for testing $\theta = \pi$ separately. [3]

B3 With $t = \tan 22.5°$, the formula gives $\tan 45° = 2t/(1 - t^2) = 1$, so $2t = 1 - t^2$, that is $t^2 + 2t - 1 = 0$. Solving gives $t = (-2 \pm \sqrt{8})/2 = -1 \pm \sqrt{2}$. Since 22.5° is acute, t is positive, so $t = \sqrt{2} - 1 \approx 0.414$. M1 for the t-formula with $\theta = 45°$, A1 for $t^2 + 2t - 1 = 0$, M1 for solving the quadratic, A1 for $\sqrt{2} - 1$ with the sign justified. [4]

SECTION C · Stretch

C1 Write the left side over $\sin\theta$ and $\cos\theta$: $(1 + 1/\sin\theta)/(\cos\theta/\sin\theta) = (\sin\theta + 1)/\cos\theta$. Now substitute the t-formulae. The numerator becomes $(2t + 1 + t^2)/(1 + t^2) = (1 + t)^2/(1 + t^2)$ and the denominator $(1 - t^2)/(1 + t^2) = (1 - t)(1 + t)/(1 + t^2)$. Dividing, the $(1 + t^2)$ and one factor of $(1 + t)$ cancel, leaving $(1 + t)/(1 - t)$ as required. Cancelling $1 + t$ needs $t \neq -1$, and that value is already excluded: $t = -1$ makes $\cot\theta$ zero, so the left side is undefined there and the identity is being proved only where both sides exist. M1 for putting the left side over a common denominator, A1 for $(\sin\theta + 1)/\cos\theta$, M1 for substituting the t-formulae, A1 for the numerator, A1 for the denominator, A1 for the cancelled result with $t \neq -1$ recorded. [6]

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