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QUESTION WORKBOOK

The trapezium rule

Numerical methods · Year 13

16 questions · 68 marks

NAVY SCREEN EDITION

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SECTION A · Warm-up

A1
TRA-A1

A student applies the trapezium rule to $\int_0^3 f(x)\,dx$ using 7 equally spaced ordinates. State the number of strips and the strip width h. [2]

A2
TRA-A2

Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{(1 + x^3)}\,dx$, giving your answer to 4 decimal places. [3]

A3
TRA-A3

For n strips of width h with ordinates $y_0, y_1, \ldots, y_n$, write down the left-endpoint rectangle sum L and the right-endpoint rectangle sum R, and state the condition on f under which one of them is a lower bound and the other an upper bound for the integral. [2]

A4
TRA-A4

A car pulls away from traffic lights, and its speed v m s^{-1} is recorded every 10 seconds. At t = 0, 10, 20, 30 and 40 s the readings are 0, 6.4, 11.3, 14.1 and 15.6. Estimate the distance travelled in the 40 seconds, giving your answer with its units, and state one assumption your estimate makes. [3]

SECTION B · Standard

B1
TRA-B1

Use the trapezium rule with 4 strips to estimate $\int_0^1 e^{x^2}\,dx$ to 4 decimal places, and state with a reason whether your estimate is an overestimate or an underestimate. [4]

B2
TRA-B2

The trapezium rule with 4 strips gives 3.2833 for $\int_0^2 \sqrt{(1 + x^3)}\,dx$, and the true value is 3.2413 to 4 decimal places. State how the estimate can be improved, find the estimate with 8 strips to 4 decimal places, and describe what has happened to the error. [4]

B3
TRA-B3

A student estimating $\int_0^3 f(x)\,dx$ with 6 strips writes down the ordinates at $x = 0, 0.5, 1, 1.5, 2$ and 2.5 and begins the bracket. Explain the mistake, state how many ordinates the rule needs here, and state how the ordinates and the strip width change if the number of strips is doubled. [3]

B4
TRA-B4

Using 4 strips, find the left-endpoint and right-endpoint rectangle sums L and R for $\int_0^1 e^{-x^2}\,dx$, giving each to 4 decimal places. State which is the upper bound and which the lower, with the condition that makes them bounds, and show that the trapezium estimate from the same ordinates is the mean of the two. [5]

B5
TRA-B5

The current I amperes through a component is recorded every 5 seconds. At t = 0, 5, 10, 15, 20, 25 and 30 s the readings are 2.0, 2.6, 3.1, 3.4, 3.5, 3.5 and 3.4. The charge that passes is the area under current against time. Estimate the charge that passes in the 30 seconds, with its units, and explain why the two rectangle sums from this table cannot be quoted as bounds for it. [4]

B6
TRA-B6

A function f is increasing on $[0, 3]$, with $f(0) = 2$ and $f(3) = 8$, and $\int_0^3 f(x)\,dx$ is to be bounded by rectangle sums with n strips. Show that the two bounds differ by $h(y_n - y_0)$, find the difference when $n = 6$, and find the least n for which the bounds are no more than 0.1 apart. State how close to the integral the trapezium estimate with that n is then guaranteed to be. [6]

B7
TRA-B7

Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{(4 - x)}\,dx$ to 4 decimal places, and state with a reason whether the estimate is above or below the true value. Then find the exact value by integration and confirm your answer. [5]

B8 TRA-B8

The table gives five values of y for a curve $y = f(x)$, each correct to 1 decimal place. No equation for f is known. Use the trapezium rule with four strips to estimate $\int_0^2 y\,dx$. Use the second differences of the tabulated values to decide whether the curve is convex or concave on this interval, and hence state, with a reason, whether your estimate is an overestimate or an underestimate. [5]

x	y
0	5.0
0.5	4.2
1.0	3.6
1.5	3.2
2.0	3.0

SECTION C · Stretch

C1
TRA-C1

A pond is surveyed by measuring its width at right angles to a base line at 10 m intervals. At distances 0, 10, 20, 30, 40, 50 and 60 m along the line the widths are 0, 24, 37, 42, 40, 31 and 0 metres. The pond has a roughly uniform depth of 2.5 m. Estimate the surface area of the pond and the volume of water it holds, giving each with its units, state the assumptions the estimates rest on, and explain why the survey cannot settle whether the area estimate is too large or too small. [6]

C2
TRA-C2

Use the trapezium rule with 4 strips to estimate $\int_0^2 1/(1 + x^2)\,dx$ to 4 decimal places. A student writes: $f''(2)$ is positive, so the curve is convex and the estimate is an overestimate. Explain why the argument fails, and, given that the exact value is 1.1071 to 4 decimal places, state whether the estimate is in fact too large or too small. [5]

C3 TRA-C3

$f(x) = xe^{-x}$, and $\int_0^3 f(x)\,dx = 1 - 4e^{-3} = 0.8009$ to 4 decimal places. Using 3 strips, find the left-endpoint and right-endpoint rectangle sums, explain why neither is a bound for the integral, and use the same ordinates to find two values between which the integral must lie. [5]

C4 TRA-C4

Oil leaks from a damaged tank, and the rate of leakage R litres per minute is recorded every 15 minutes. At t = 0, 15, 30, 45, 60, 75 and 90 minutes the readings are 3.2, 2.7, 2.3, 2.0, 1.8, 1.7 and 1.6. Estimate the volume of oil lost in the 90 minutes, with its units and the assumption it rests on. An engineer says that, because the readings decrease throughout, the two rectangle sums bound the volume: state the extra assumption that claim needs, and find the two sums. State how the estimate could be improved, and how far apart the two sums would then be if readings were taken every 5 minutes and the first and last readings were unchanged. [6]

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