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ANSWER BOOK

Triangles and the sine and cosine rules

Trigonometry · Year 12–13

7 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The known opposite pair is a and A, so the sine rule gives b = 10 sin 80°/sin 35° = 17.2 cm. M1 for the sine rule, A1 for 17.2 cm. The bigger angle faces the bigger side, and 17.2 > 10 respects that, which is a free sanity check. Pairing the 80° with the 10 cm instead is the usual slip, and it returns 5.82 cm rather than 17.2. [2]

A2 Area = ½ × 8 × 11 × sin 30° = 44 × ½ = 22 cm^2. M1 for ½ab sin C, A1 for the exact area. The angle has to be the one squeezed between the two sides used, so an included angle is what the formula demands. Exact here means no rounding is needed at all, since sin 30° = ½. [2]

SECTION B · Standard

B1 Two sides and the included angle call for the cosine rule. a^2 = 36 + 81 - 2 × 6 × 9 × cos 40° = 117 - 82.73... = 34.27..., so a = 5.85 cm. M1 for the cosine rule, A1 for 34.27, A1 for 5.85 cm. Hold a^2 at full accuracy until the final square root; rounding 34.27 to 34 costs the third figure. Note the whole product 2bc cos A is subtracted, rather than cos A on its own. [3]

B2 The largest angle faces the longest side, so it is the one opposite the 7 cm. The rearranged cosine rule gives cos A = (25 + 36 - 49)/(2 × 5 × 6) = 12/60 = 0.2, so A = 78.5° to 1 decimal place. B1 for choosing the angle opposite the 7 cm, M1 for the rearranged cosine rule, A1 for 78.5°. A positive cosine means an acute angle, so even the biggest angle here is under 90°. Choosing the wrong side to be opposite is what this question tests, and it is worth a mark before any arithmetic happens. [3]

B3 First the third angle: B = 180° - 50° - 60° = 70°. Then, using the known opposite pair a and A, b = 12 sin 70°/sin 50° = 14.7 cm and c = 12 sin 60°/sin 50° = 13.6 cm. B1 for B = 70°, M1 for the sine rule, A1 for b = 14.7 cm, M1 for the second application, A1 for c = 13.6 cm. The largest side, b, faces the largest angle, B, as it must. Solve the triangle means every unknown part, so an answer that stops after one side leaves marks on the table. [5]

SECTION C · Stretch

C1 Sine rule: sin B = 9 sin 40°/7 = 0.8264..., so B = 55.7° or its supplement 124.3°. Both must be tested against the angle sum: 40° + 124.3° = 164.3° < 180°, so the obtuse option also leaves room for C. M1 for the sine rule, A1 for sin B = 0.8264, A1 for 55.7°, M1 for taking the supplement, A1 for 124.3°. Two triangles exist: sine cannot tell an angle from its supplement, and only the angle sum can referee. [5]

C2 Cosine rule with $\cos 60° = \frac{1}{2}$: $a^2 = 16 + 36 - 48 \times \frac{1}{2} = 28$, so $a = \sqrt{28} = 2\sqrt{7}$ cm. [5]
Area $= \frac{1}{2} \times 4 \times 6 \times \sin 60° = 12 \times \sqrt{3}/2 = 6\sqrt{3}$ cm^2. M1 for the cosine rule with $\cos 60° = \frac{1}{2}$, A1 for 28, A1 for $2\sqrt{7}$ cm, M1 for $\frac{1}{2}ab \sin C$, A1 for the exact area. The 60° angle is what makes exact work possible: its sine and cosine are both on the exact-value list.