inkMaths

ANSWER BOOK

Vectors in three dimensions

Vectors · Year 12–13

7 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $|6i - 2j + 3k| = \sqrt{(36 + 4 + 9)} = \sqrt{49} = 7$. M1 for the sum of three squares, A1 for 7. [2]
Three squares under one root, which is the two-dimensional formula with one more term. The formula is Pythagoras applied twice. Run it first across the floor of the box, where the diagonal of the base rectangle has length $\sqrt{(36 + 4)} = \sqrt{40}$, then again up the vertical, on the right-angled triangle made by that diagonal and the k-component, giving $\sqrt{(40 + 9)} = 7$.

A2 Since $|a| = 7$, the unit vector is $(6/7)i - (2/7)j + (3/7)k$. For magnitude 21, scale by $21/7 = 3$, giving $18i - 6j + 9k$, or its negative $-18i + 6j - 9k$, since parallel allows either sense. B1 for the unit vector, M1 for scaling by 3, A1 for the vector of magnitude 21. A question asking for the same direction as a would accept only the first of those. [3]

SECTION B · Standard

B1 $AB = b - a = 2i - 4j + 6k$. The distance is $|AB| = \sqrt{(4 + 16 + 36)} = \sqrt{56} = 2\sqrt{14}$, which the word exact requires you to leave in surd form rather than write as 7.48. The midpoint averages the coordinates, giving (2, 0, 2). M1 for $b - a$, A1 for $2i - 4j + 6k$, A1 for $2\sqrt{14}$, B1 for the midpoint. Averaging -1 and 5 to get 2 is where the negative coordinate usually claims a victim. [4]

B2 $a^2 + 16 + 64 = 144$, so $a^2 = 64$ and $a = \pm 8$. M1 for the magnitude equation, A1 for $a = 8$, A1 for $a = -8$. Both signs survive, because squaring hides the sign, so the answer is two vectors that are mirror images in the plane $x = 0$. Giving only $a = 8$ loses the accuracy mark on a question that says values, plural. [3]

B3 $PQ = 2i + 4j + 2k$ and $PR = 5i + 10j + 5k$, so $PR = 2.5PQ$, a scalar multiple. The two vectors are therefore parallel, and they share the point P, so P, Q and R lie on one line. M1 for two of the vectors, A1 for the scalar multiple, B1 for the common point. In three dimensions there is no single cross-multiplication test, so the same multiplier has to work in all three components. Stating that the vectors are parallel without naming the common point leaves the proof incomplete. [3]

SECTION C · Stretch

C1 AB = i + 2j + 2k, so |AB| = √(1 + 4 + 4) = 3, and AC = 2i - 2j + k, so |AC| = √(4 + 4 + 1) = 3. Two equal sides makes it isosceles. For the right angle, BC = c - b = i - 4j - k, so $|BC|^2$ = 1 + 16 + 1 = 18. Since $|AB|^2 + |AC|^2$ = 9 + 9 = 18 = $|BC|^2$, the converse of Pythagoras places the right angle at A, between the two equal sides. M1 for AB and AC, A1 for the two equal lengths, B1 for isosceles, M1 for BC, A1 for $|BC|^2$ = 18, A1 for the converse of Pythagoras. Test the longest side as the hypotenuse. Trying the relation on AB or AC instead produces a false statement and wastes the working. [6]

C2 The two sides meeting at the right angle are AB and AC, and they serve as base and perpendicular height. |AB| = √(1 + 4 + 4) = 3 and |AC| = √(4 + 4 + 1) = 3, so the area is ½ × 3 × 3 = 9/2. B1 for using AB and AC as base and height, M1 for ½ × 3 × 3, A1 for 9/2. The right angle is what makes this legitimate. Without it, the product of two side lengths says nothing about the area, and |BC| = √18 is the hypotenuse, so using it as a base would need a perpendicular height that has not been found. [3]