InkMaths

ANSWER BOOK

Vectors in two dimensions

Vectors · Year 12–13

7 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $|\mathbf{a}| = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$. The unit vector is $\mathbf{a}$ divided by its own length, $(5/13)\mathbf{i} + (12/13)\mathbf{j}$. B1 for 13, B1 for the unit vector. Its magnitude is 1 by construction, which makes a quick self-check. Dividing by 13 must reach both components, and halving that job is the usual slip. [2]

A2 $2\mathbf{p} + 3\mathbf{q} = (4 - 3)\mathbf{i} + (-6 + 12)\mathbf{j} = \mathbf{i} + 6\mathbf{j}$. M1 for multiplying out both brackets, A1 for $\mathbf{i} + 6\mathbf{j}$. The arithmetic runs slot by slot, and the only traps are the signs. Multiply through each bracket before collecting. [2]

A3 $|7\mathbf{i} - 24\mathbf{j}| = \sqrt{(49 + 576)} = \sqrt{625} = 25$. The angle satisfies $\tan\theta = 24/7$, so $\theta = 73.7°$, measured below the positive x-direction because the j-component is negative. B1 for 25, M1 for $\tan\theta = 24/7$, A1 for the angle below the axis. Stating which side of the axis the vector lies is part of the answer, since a magnitude and a bare angle do not pin a vector down. [3]

SECTION B · Standard

B1 $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = (4 - (-2))\mathbf{i} + (-5 - 3)\mathbf{j} = 6\mathbf{i} - 8\mathbf{j}$, destination minus start. The distance is $|AB| = \sqrt{(36 + 64)} = \sqrt{100} = 10$. The midpoint averages the position vectors, giving $((-2 + 4)/2, (3 - 5)/2) = (1, -1)$. M1 for $\mathbf{b} - \mathbf{a}$, A1 for $6\mathbf{i} - 8\mathbf{j}$, A1 for the distance, B1 for the midpoint. Writing $\mathbf{a} - \mathbf{b}$ instead of $\mathbf{b} - \mathbf{a}$ reverses the vector and costs the first mark, though the distance survives. [4]

B2 Parallel means one vector is a scalar multiple of the other, so $6\mathbf{i} + \lambda\mathbf{j} = k(4\mathbf{i} + 6\mathbf{j})$. Comparing i-components gives $k = 6/4 = 3/2$, and then $\lambda = 6 \times 3/2 = 9$. Equivalently the components are in proportion, $4/6 = 6/\lambda$, which cross-multiplies to $4\lambda = 36$. M1 for setting up the scalar multiple, A1 for $k = 3/2$, A1 for $\lambda = 9$. Setting up the proportion upside down gives $\lambda = 4$, so check the answer against the pattern that both components grow by the same factor. [3]

B3 $|5\mathbf{i} - 12\mathbf{j}| = \sqrt{(25 + 144)} = 13$, and $26 = 2 \times 13$, so scale by ± 2. The vectors are $10\mathbf{i} - 24\mathbf{j}$ and $-10\mathbf{i} + 24\mathbf{j}$. M1 for the magnitude 13 and the scale factor 2, A1 for $10\mathbf{i} - 24\mathbf{j}$, A1 for $-10\mathbf{i} + 24\mathbf{j}$. Forgetting the negative one loses a mark, because parallel includes pointing the opposite way. A question asking for the same direction rather than parallel would want only the first. [3]

SECTION C · Stretch

C1 P is one third of the way from A to B, so OP = a + ⅓(b − a). Here b − a = 15i + 6j, so ⅓(b − a) = 5i + 2j and OP = (1 + 5)i + (6 + 2)j = 6i + 8j. Then |OP| = √(36 + 64) = 10. M1 for OP = a + ⅓(b − a), A1 for ⅓(b − a), A1 for 6i + 8j, M1 for the magnitude, A1 for 10. Check the ratio afterwards. AP = 5i + 2j and PB = 10i + 4j, which is 2AP, so AP : PB = 1 : 2 as required. The ratio 1 : 2 splits AB into three equal parts, not two, and reading it as a half is the error that dominates this question. The other trap is applying the fraction to b rather than to b − a, which gives a point that does not lie on AB at all. The reliable form is OP = a + [m/(m + n)](b − a) for a ratio m : n. [5]