



ANSWER BOOK

Volumes of revolution

Further calculus · Year FM

7 questions · 26 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $V = \pi \int y^2 dx$ from a to b . Each strip sweeps a disc of radius y and thickness dx , [2]
and a disc's volume is π times radius squared times thickness. B1 for the formula, B1 for the disc explanation.
- A2** $V = \pi \int 9 dx$ from 0 to 4 = 36π . The solid is a cylinder of radius 3 and length 4, [3]
and $\pi r^2 h = \pi \times 9 \times 4 = 36\pi$ agrees. M1 for $\pi \int 9 dx$, A1 for 36π , B1 for the cylinder check.
Constant y is the quickest sanity check the formula ever gets.

SECTION B · Standard

- B1** $V = \pi \int x^4 dx$ from 0 to 2 = $\pi[x^5/5] = 32\pi/5$. Square first, integrate second. M1 for [4]
squaring y , A1 for the integrand, M1 for integrating, A1 for $32\pi/5$. The integrand is x^4 , not x^2 , and the power-rule slip of integrating y before squaring is the topic's classic error.
- B2** Radius is x with $x^3 = y$, so $x^2 = y^{2/3}$ and $V = \pi \int y^{2/3} dy$ from 0 to 8 = $\pi[3y^{5/3}/5] = \pi \times$ [4]
 $3 \times 32/5 = 96\pi/5$. M1 for x^2 in terms of y , A1 for the integrand, M1 for integrating with the y -limits, A1 for $96\pi/5$. About the y -axis everything transposes. The limits become y -limits, and x^2 has to be expressed in terms of y .
- B3** Outer radius x , inner radius x^2 : $V = \pi \int (x^2 - x^4) dx$ from 0 to 1 = $\pi(1/3 - 1/5) = 2\pi/15$. [4]
M1 for the difference of squares, A1 for the integrand, M1 for integrating, A1 for $2\pi/15$.
Subtract the squares, never square the difference: $\pi \int (x - x^2)^2 dx$ describes a different and wrong solid.
- B4** Each cross-section is an annulus, a disc of the outer radius with a disc of the [3]
inner radius removed, of area $\pi(\text{outer}^2 - \text{inner}^2)$. Squaring the difference instead computes a disc whose radius is the gap between the curves, which is a different region rotated about a different axis. B1 for the annulus, B1 for its area, B1 for what squaring the difference computes instead.

SECTION C · Stretch

- C1** The formula $V = \pi \int y^2 dx$ still applies, with dx rewritten in terms of t . Here $dx/dt =$ [6]
 $2t$, so $dx = 2t dt$, and $y^2 = 4t^2$. The limits transform as well, since $x = 0$ gives $t = 0$ and $x = 4$ gives $t = 2$. So $V = \pi \int 4t^2 \times 2t dt$ from 0 to 2 = $\pi \int 8t^3 dt = \pi[2t^4]$ from 0 to 2 = 32π . M1 for $dx = 2t dt$, B1 for the new limits, M1 for $y^2 = 4t^2$, A1 for $\pi \int 8t^3 dt$, M1 for integrating, A1 for 32π .
As a check, eliminating t gives $y^2 = 4x$, and $\pi \int 4x dx$ from 0 to 4 = $\pi[2x^2] = 32\pi$ as well.
Leaving the limits in x while integrating with respect to t is the error that costs most marks on this type.