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ANSWER BOOK

Forced vibrations and resonance

Periodic motion · Year 13

AQA 3.6.1.4 · CIE 17.3.3 · OCR A 5.3.3(a), 5.3.3(b)(i), 5.3.3(c), 5.3.3(d), 5.3.3(e)

18 questions · 54 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Free vibrations occur when a system is displaced and released, oscillating at its own natural frequency with no driving force (1). Forced vibrations occur when a periodic external driving force makes the system oscillate at the driving frequency (1). [2]

A2 Resonance is the large-amplitude response of a system when it is driven at (or very close to) its natural frequency (1), so that energy is transferred to it most efficiently (1). [2]

A3 The driving frequency must equal the natural frequency of the system (1), or be very close to it (1). [2]

A4 The pendulum whose length is also 0.60 m (1). Its natural frequency equals the driving frequency of the driver pendulum, so it resonates (1). [2]

A5 The sound waves from the first fork act as a periodic driving force on the second fork at 512 Hz (1). This equals the natural frequency of the identical second fork, so it resonates and oscillates with large amplitude (1). [2]

SECTION B · Standard

B1 $f = (1/2\pi)\sqrt{(k/m)}$ (1) [3]
$= (1/2\pi)\sqrt{(100/0.25)} = 3.18$ Hz (1)
Resonance occurs when the driving frequency equals this, about 3.18 Hz (1)

B2 With light damping the graph rises to a sharp peak at the natural frequency and falls away on either side (1). At low frequencies the amplitude is small (1); it grows to a maximum at resonance, then decreases again at higher frequencies (1). [3]

B3 $T = 2\pi\sqrt{(l/g)} = 2\pi\sqrt{(0.50/9.81)} = 1.42$ s (1) [3]
$f = 1/T = 0.70$ Hz (1)
Resonance occurs when driven at about 0.70 Hz (1)

B4 More damping lowers the resonance peak, giving a smaller maximum amplitude (1). It also broadens the peak over a wider range of frequencies (1) and shifts it to a slightly lower frequency (1). [3]

B5 The engine provides a periodic driving force at 1500/60 = 25 Hz (1) [3]
Violent vibration means resonance, so the natural frequency of the mirror is 25 Hz (1)
At 3000 rpm the driving frequency (50 Hz) is well above the natural frequency, so the amplitude is small and the mirror is nearly steady (1)

B6 For the fundamental, $\lambda = 2L = 1.6$ m (1) [3]
$f = v/\lambda = 40/1.6 = 25$ Hz (1)
A large-amplitude stationary wave forms on the string: the string resonates (1)

B7 The natural frequency is about 2.5 Hz, where the amplitude peaks (1). At 2.0 Hz and 3.0 Hz the driving frequency does not match the natural frequency, so the pushes fall out of step with the motion (1). Energy is then transferred to the system far less efficiently, and some pushes partly cancel the motion, so the amplitude stays small (1). [3]

SECTION C · Stretch

C1 Well below 50 Hz the amplitude is small (1). As the driving frequency approaches 50 Hz the amplitude grows rapidly, reaching a maximum at resonance (1). With light damping that maximum sits at 50 Hz; heavier damping moves it a little below 50 Hz and flattens it (1). As the frequency rises further above 50 Hz the amplitude falls again, becoming small at high frequencies (1). [4]

C2 If a periodic force (such as wind or marching feet) matches the structure's natural frequency (1), resonance builds up large-amplitude oscillations (1) that can cause damage or collapse (1). Engineers add damping (for example tuned mass dampers) to absorb energy and limit the amplitude, or design the structure so its natural frequency avoids likely driving frequencies (1). [4]

C3 Useful: the tuning circuit of a radio resonating at the chosen station's frequency (or MRI, or a pushed swing) (1). Problematic: a bridge or building oscillating in the wind, or a washing machine vibrating badly at certain spin speeds (1). [2]

C4 Each ridge gives the suspension one push, so the driving frequency is v/d, where $d = 12$ m (1) [4]
Resonance needs $v/d = 1.3$ Hz, so $v = 1.3 \times 12 = 15.6$ m s^{-1} (1)
15.6 m s^{-1} is above the 13.4 m s^{-1} limit (1)
So within the speed limit the driving frequency stays below the natural frequency and resonance cannot occur (1)

C5 Well below the natural frequency the oscillator moves in phase with the driver (1). At the natural frequency the oscillator lags the driver by a quarter of a cycle (90°), and by exactly that whatever the damping (1). Well above the natural frequency the oscillator is in antiphase with the driver, lagging by half a cycle (180°) (1). [3]

C6 The panel is an oscillator with its own natural frequency (1). The rotating motor applies a periodic driving force to the panel at the rotation frequency, so the panel performs forced vibrations (1). When the motor speed brings the driving frequency close to the panel's natural frequency, the panel resonates (1). At resonance energy is transferred from the motor to the panel at the maximum rate, so the amplitude becomes large and the panel rattles; away from this band the amplitude is small (1). Adding damping, for example a rubber or self-adhesive damping layer on the panel, lowers and broadens the resonance peak so the maximum amplitude is much smaller (1). Alternatively, stiffening the panel or adding mass changes its natural frequency so that it no longer lies within the motor's running range (1). [6]