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ANSWER BOOK

Heat engines and heat pumps

Engineering physics · Year 13

AQA 3.11.2.4, 3.11.2.5, 3.11.2.6

17 questions · 51 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Each cycle the working substance absorbs heat Q_H from a hot source, converts part of it to useful work W, and rejects the rest, Q_C, to a cold sink (1). Energy is conserved, so $W = Q_H - Q_C$ (1). [2]

A2 The working substance must be returned to its starting state to repeat the cycle, which means compressing it, and that requires heat Q_C to be rejected to a colder sink (1). Since Q_C can never be zero, $W = Q_H - Q_C$ is always less than Q_H, so the efficiency is always below 1 (1). [2]

A3 Maximum theoretical efficiency = $(T_H - T_C)/T_H$ (1), where T_H and T_C are the source and sink temperatures in **kelvin** (1). It is a ratio of absolute temperatures, so a celsius value anywhere in it voids the answer. [2]

A4 Refrigerator: COP = Q_C/W, the heat removed from the cold space per unit of work input (1). Heat pump: COP = Q_H/W, the heat delivered to the hot space per unit of work input (1). Each is a benefit divided by the work paid for, and each is a pure number, not a percentage. [2]

A5 The net work per cycle is the area enclosed by the loop (1), and an engine goes round it clockwise, expanding at high pressure and compressing at low pressure so that more work comes out than goes back in (1). [2]

A6 Input power = calorific value of the fuel × the rate at which fuel is burned (1), in J kg^{-1} × kg s^{-1} to give watts. [1]

SECTION B · Standard

B1 $W = Q_H - Q_C = 1500 - 1050 = 450$ J (1). Efficiency = W/Q_H (1) = 450/1500 = 0.30, that is 30% (1). Dividing W by Q_C instead of Q_H is the standard slip; the denominator is always what went in. [3]

B2 Convert to kelvin first: $T_H = 520 + 273 = 793$ K and $T_C = 30 + 273 = 303$ K (1). Maximum efficiency = $(T_H - T_C)/T_H$ (1) = (793 - 303)/793 = 0.62, about 62% (1). Working in celsius gives 0.94, wildly too high, and is the single most common error in this topic. [3]

B3 Indicated power = area of loop × cycles per second × number of cylinders (1) = 260 × 30 × 4 (1) = 31200 W, that is 31.2 kW (1). Forgetting to multiply by the cylinder count is the standard slip and gives a quarter of the true power. [3]

B4 $\omega = 3600 \times 2\pi/60 = 377$ rad s^{-1} (1). Brake power = $T\omega$ (1) = $145 \times 377 = 5.47 \times 10^4$ W, about 55 kW (1). $P = T\omega$ needs ω in rad s^{-1}, so the rev min^{-1} conversion has to come first. [3]

B5 Input power = calorific value × fuel flow rate = $43 \times 10^6 \times 3.5 \times 10^{-3}$ (1) = 1.50×10^5 W, about 150 kW (1). Overall efficiency = brake power/input power = 52/150 = 0.35 (1). Converting MJ kg^{-1} to J kg^{-1} is the step that is most often missed. [3]

B6 $COP_{ref} = Q_C/W$ (1) = 250/100 = 2.5 (1). $Q_H = Q_C + W = 250 + 100 = 350$ J (1). A COP is a plain number, so writing it as 250% is wrong: it is not an efficiency and it is expected to exceed 1. [3]

B7 $COP_{hp} = Q_H/W$, so $W = Q_H/COP$ (1) = 7.2/3.6 = 2.0 kW (1). $Q_C = Q_H - W = 7.2 - 2.0 =$ 5.2 kW drawn from the cold outdoor air (1). Multiplying by the COP instead of dividing is the common error here. [3]

SECTION C · Stretch

C1 Input power = $45 \times 10^6 \times 4.0 \times 10^{-3}$ = 180 kW (1). Indicated power = 480 × 25 × 6 = 72 kW (1). $\omega = 2900 \times 2\pi/60 = 304$ rad s^{-1} (1), so brake power = $T\omega$ = 210 × 304 = 63.8 kW (1). Friction power = indicated - brake = 72 - 63.8 = 8.2 kW (1). Overall efficiency = brake/input = 63.8/180 = 0.35 (1). Check the ladder descends: 180 kW in, 72 kW developed in the cylinders, 63.8 kW at the shaft. An answer with brake power above indicated power has slipped, usually by dropping the cylinder count from the indicated power. [6]

C2 $W = Q_H - Q_C = 2400 - 1560 = 840$ J per cycle (1). Power = 840 × 20 = 1.68×10^4 W, that is 16.8 kW (1). Efficiency = W/Q_H = 840/2400 = 0.35 (1). Maximum = (900 - 320)/900 = 0.64 (1), the temperatures already being in kelvin. The engine reaches 0.35 against a ceiling of 0.64, a little over half of what is theoretically available; the shortfall is friction, turbulence and heat leaking out of the cylinder, none of which the ideal ceiling allows for (1). An efficiency above the ceiling always means an arithmetic error, usually a temperature left in celsius. [5]

C3 Electric heater: essentially all the electrical power becomes heat, so it delivers 3.0 kW (1). Heat pump: $Q_H = COP \times W = 4.0 \times 3.0 = 12$ kW (1), four times as much. Conservation holds because 9 kW of that is heat extracted from the cold outdoor air and merely moved indoors; only 3.0 kW is created from electrical work (1). The pump transfers heat rather than generating it, which is why a COP above 1 is normal (1). Its COP falls on a colder day, because the temperature gap it has to pump across is then wider. [4]

C4 $COP_{ref} = Q_C/W$, so $Q_C = 2.4 \times 90 = 216$ W (1). $Q_H = Q_C + W = 216 + 90 = 306$ W delivered to the kitchen (1). With the door open the heat removed from the compartment simply returns to the kitchen, so the room receives 306 W while losing 216 W (1), a net gain of 90 W, exactly the electrical work the compressor is doing, which ends up as heat (1). Treating the COP as a percentage, or forgetting that Q_H exceeds Q_C by the work input, are the two common errors. [4]

