InkPhysics

ANSWER BOOK

Orbits and satellites

Gravitational fields · Year 13

AQA 3.7.2.4 · CIE 13.2.3, 13.2.4 · OCR A 5.4.3(a), 5.4.3(b), 5.4.3(c), 5.4.3(d), 5.4.3(e), 5.4.4(e)

19 questions · 53 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The gravitational attraction of the planet on the satellite (1); directed towards the centre of the planet (the centre of the orbit) (1). [2]

A2 $T^2 \propto r^3$: the square of the orbital period is proportional to the cube of the orbital radius (1). [1]

A3 Any two of the following, one mark each (2): period of 24 hours, equal to the Earth's rotation period; orbit is circular and lies in the equatorial plane; satellite moves west to east, in the same sense as the Earth's rotation, so it stays above a fixed point on the equator. [2]

A4 The orbital speed decreases, since $v = \sqrt{(GM/r)}$ (1); the orbital period increases (1). [2]

A5 In $GMm/r^2 = mv^2/r$ the satellite's mass m cancels, leaving $v = \sqrt{(GM/r)}$ (1). [1]

A6 $T^2 \propto r^3$, so $T_B/T_A = 4^{3/2}$ (1) [2]
$T_B/T_A = 8$ (1)

SECTION B · Standard

B1 $GMm/r^2 = mv^2/r$, so $v = \sqrt{(GM/r)}$ (1) [3]
$v = \sqrt{(6.67 \times 10^{-11} \times 6.0 \times 10^{24}/(7.0 \times 10^{6}))}$ (1)
$v = 7561 \text{ m s}^{-1}$ (1)

B2 $T = 2\pi r/v = (2\pi \times 7.0 \times 10^{6})/7561$ (1) [2]
$T = 5817$ s (about 97 minutes) (1)

B3 $T^2 = 4\pi^2 r^3/GM$, so $r = (GMT^2/4\pi^2)^{1/3}$ (1) [3]
$r = ((6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 86400^2)/4\pi^2)^{1/3}$ (1)
$r = 4.23 \times 10^{7}$ m (1)

B4 Low orbit: altitude of a few hundred kilometres with a period of about 90 minutes; geostationary: altitude of about 36 000 km with a period of 24 hours (1). Low orbits are used for Earth imaging (or the space station), as they pass close to the surface (1). Geostationary orbits are used for communications or weather satellites, which must remain above a fixed point on the equator (1). [3]

B5 $GMm/r^2 = mv^2/r$ (gravitational force provides the centripetal force) (1) [3]
Substitute $v = 2\pi r/T$: $GM/r = 4\pi^2r^2/T^2$ (1)
Rearranging gives $T^2 = 4\pi^2r^3/GM$ (1)

B6 T^2/r^3 is the same for both moons (both orbit the same central mass) (1) [3]
$T_2 = 27 \times (1.0/4.0)^{3/2}$ (1)
$T_2 = 3.4$ days (1)

B7 Orbital speed: $v_{orb} = \sqrt{(GM/r)}$, from gravity acting as the centripetal force (1) [3]
Escape velocity: $v_{esc} = \sqrt{(2GM/r)}$ (1)
$v_{esc}/v_{orb} = \sqrt{2}$, whatever the value of r (1)

SECTION C · Stretch

C1 $v = \sqrt{(GM/r)} = 7073\ \text{m s}^{-1}$ (1) [4]
$KE = ½mv^2 = 1.25 \times 10^{10}$ J (1)
$PE = -GMm/r = -2.50 \times 10^{10}$ J (1)
Total = KE + PE = -1.25×10^{10} J (1)

C2 $r = (GMT^2/4\pi^2)^{1/3}$ (1) [3]
$r = ((6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (1.0 \times 10^5)^2)/4\pi^2)^{1/3}$ (1)
$r = 4.66 \times 10^7$ m (1)

C3 The plane of any orbit must contain the centre of the Earth (1). To remain above a fixed point as the Earth rotates, the satellite's orbital plane must coincide with the equatorial plane (1); an orbit inclined to the equator would carry the satellite north and south of the point, so it would not appear stationary (1). [3]

C4 For moons of one planet, T^2/r^3 should be the same for all (1) [4]
In units of days^2 per $(10^8\ \text{m})^3$: W gives 4.0, X gives 4.1, Y gives 4.0 (1)
Z gives 3.0 (1)
W, X and Y agree; moon Z's data are inconsistent with Kepler's third law, so Z's measurement is the suspect one (1).

C5 Drag transfers energy away from the satellite, so its total energy falls (1); the satellite therefore spirals down to a smaller orbital radius (1); the orbital speed $v = \sqrt{(GM/r)}$ increases as r decreases, so the kinetic energy rises (1); the potential energy falls by more than the kinetic energy rises, giving the net loss of total energy (1). [4]

C6 In a circular orbit KE = $\frac{1}{2}mv^2$ = GMm/2r and PE = -GMm/r, so the total energy is [5]
E = -GMm/2r (1)
E_1 = -GMm/(2 × 9.0 × 10^6) = -1.33 × 10^{10} J (1)
E_2 = -GMm/(2 × 1.8 × 10^7) = -6.67 × 10^9 J (1)
Energy supplied = E_2 - E_1 (1)
= 6.67 × 10^9 J (1)