inkPhysics

ANSWER BOOK

SHM systems: pendulums and springs

Periodic motion · Year 13

AQA 3.6.1.3 · CIE 17.2.1, 17.2.2, 17.3.1, 17.3.2 · OCR A 5.3.1(c)(ii), 5.3.2(a), 5.3.2(b), 5.3.3(b)(i)

19 questions · 56 marks

NAVY EDITION

SECTION A · Warm-up

A1 $T = 2\pi\sqrt{(m/k)}$ (1), where T is the period, m the oscillating mass and k the spring constant (1). [2]

A2 $T = 2\pi\sqrt{(l/g)} = 2\pi\sqrt{(1.0/9.81)}$ (1)
$T = 2.01$ s (1) [2]

A3 They interchange: kinetic energy is greatest at the equilibrium position (where potential energy is least) (1), and potential energy is greatest at the extremes (where kinetic energy is zero) (1). The total energy stays constant. [2]

A4 The oscillations must be of small amplitude (small angle of swing) (1). [1]

A5 The period doubles, because $T \propto \sqrt{m}$ (1). [1]

A6 The mass of the bob (1); and the amplitude, provided the swings are small (1). [2]

SECTION B · Standard

B1 $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(0.50/200)}$ (1)
$T = 0.314$ s (1) [2]

B2 From $T = 2\pi\sqrt{(l/g)}$: $l = g(T/2\pi)^2$ (1)
$= 9.81 \times (2.0/2\pi)^2$ (1)
$l = 0.994$ m (1) [3]

B3 (a) $k = 4\pi^2 m/T^2 = 4\pi^2 \times 0.20/0.80^2$ (1)
$k = 12.34\ \text{N m}^{-1}$ (1)
(b) $f = 1/T = 1/0.80$ (1)
$f = 1.25$ Hz (1) [4]

B4 Total energy = maximum elastic PE = $\frac{1}{2}kA^2$ (1)
$= \frac{1}{2} \times 50 \times 0.10^2$ (1)
$E = 0.25$ J (1) [3]

B5 $T = 35.9/20 = 1.795$ s (1)
From $T = 2\pi\sqrt{(l/g)}$: $g = 4\pi^2 l/T^2 = 4\pi^2 \times 0.800/1.795^2$ (1)
$g = 9.80\ \text{m s}^{-2}$ (1) [3]

B6 At equilibrium $kx = mg$, so $k = mg/x = (0.30 \times 9.81)/0.045$ (1) [4]
$k = 65.4$ N m^{-1} (1)
$T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(0.30/65.4)}$ (1)
$T = 0.43$ s (1)

B7 Total energy = $½kA^2 = ½ \times 80 \times 0.060^2 = 0.144$ J (1) [3]
PE at $x = 0.030$ m: $½kx^2 = ½ \times 80 \times 0.030^2 = 0.036$ J (1)
KE = total - PE = 0.144 - 0.036 = 0.108 J (1)

SECTION C · Stretch

C1 $T = 2\pi\sqrt{(l/g)} = 2\pi\sqrt{(1.0/1.62)}$ (1) [4]
$T = 4.94$ s (1)
This is much longer than on Earth (1), because the weaker gravity gives a smaller restoring effect (1)

C2 $\omega = \sqrt{(k/m)} = \sqrt{(120/0.30)} = 20$ rad s^{-1} (1) [3]
$v_{max} = \omega A = 20 \times 0.050$ (1)
$v_{max} = 1.00$ m s^{-1} (1)

C3 Damping (resistive forces such as friction or air resistance) removes energy from the oscillator (1), so the amplitude decreases with each cycle until the motion stops (1). Light damping barely changes the period; heavy damping can stop the system oscillating at all (1). [3]

C4 New period $T = 2\pi\sqrt{(0.994/9.78)}$ (1) [5]
$T = 2.0031$ s, longer than 2.000 s (1)
Each swing takes longer, so the clock counts too few swings per day: it runs slow (1)
Time lost per day = 86 400 × (2.0031 - 2.0000)/2.0031 ≈ 132 s (accept 125 to 135 s) (1)
Shorten the pendulum (so that $2\pi\sqrt{(l/9.78)} = 2.000$ s again) (1)

C5 In both cases the graph is an oscillation whose amplitude decreases with time, since air resistance damps the motion (1). With the card the damping force is larger, so the amplitude decays much more quickly (1). The decay is roughly exponential: the amplitude falls by a similar fraction each cycle (1). The period is almost unchanged by this light damping, so the two graphs keep nearly the same spacing of peaks (1). The total mechanical energy of the oscillator decreases as work is done against the resistive force, and it falls faster with the card attached (1). The energy is transferred to the surroundings as internal energy (of the air and the system) (1). [6]

C6 $T = 2\pi\sqrt{(0.30/(2 \times 9.81))}$ (1) [3]

$T = 0.78$ s (1)

No change: density does not appear in the formula, because both the restoring force and the mass of the column scale with density, so it cancels (1)