



ANSWER BOOK

Simple harmonic motion

Periodic motion · Year 13

AQA 3.6.1.2 · CIE 17.1.1, 17.1.2, 17.1.3, 17.1.4, 17.1.5
OCR A 5.3.1(a), 5.3.1(b), 5.3.1(c)(i), 5.3.1(d), 5.3.1(e), 5.3.1(f), 5.3.1(g)

19 questions · 55 marks

NAVY EDITION

SECTION A · Warm-up

- A1** The acceleration is directly proportional to the displacement from the equilibrium position (1) and is always directed towards it: $a = -\omega^2 x$ ($a \propto -x$) (1). [2]
- A2** $\omega = 2\pi f = 2\pi \times 2.0$ (1) [2]
 $\omega = 12.57 \text{ rad s}^{-1}$ (1)
- A3** $v_{\max} = \omega A = 10 \times 0.050$ (1) [2]
 $v_{\max} = 0.50 \text{ m s}^{-1}$ (1)
- A4** $a = \omega^2 x = 7.0^2 \times 0.030 = 1.5 \text{ m s}^{-2}$ (1). It is directed towards the equilibrium position, opposite to the displacement (1). [2]
- A5** (a) At maximum displacement, $x = \pm A$, the extremes of the motion (1). (b) At zero displacement, the equilibrium position (1). [2]
- A6** $f = 50/40 = 1.25 \text{ Hz}$ (1) [2]
 $T = 1/f = 0.80 \text{ s}$ (1)

SECTION B · Standard

- B1** $\omega = 2\pi f = 2\pi \times 3.0$ (1) [3]
 $\omega = 18.85 \text{ rad s}^{-1}$ (1)
 $a_{\max} = \omega^2 A = 7.11 \text{ m s}^{-2}$ (1)
- B2** $v = \pm \omega \sqrt{A^2 - x^2}$ (1) [3]
 $= 8.0 \times \sqrt{0.10^2 - 0.060^2}$ (1)
 $v = 0.64 \text{ m s}^{-1}$ (1)
- B3** $x = A \cos \omega t = 0.15 \times \cos(4.0 \times 0.20)$ (1) [3]
 $= 0.15 \times \cos(0.80)$, with the calculator in radians (1)
 $x = 0.105 \text{ m}$ (1)
- B4** $f = \omega/2\pi = 6.28/(2\pi) = 1.00 \text{ Hz}$ (1) [2]
 $T = 1/f = 1.00 \text{ s}$ (1)
- B5** $\omega = 2\pi f = 2\pi \times 0.50 = 3.14 \text{ rad s}^{-1}$ (1) [3]
 $x = 0.12 \times \sin(3.14 \times 0.40)$, with the calculator in radians (1)
 $x = 0.114 \text{ m}$ (1)

B6 $\omega = 2\pi f = 2\pi \times 250 = 1571 \text{ rad s}^{-1}$ (1) [4]
 $a_{\max} = \omega^2 A = 1571^2 \times 3.5 \times 10^{-4}$ (1)
 $a_{\max} = 864 \text{ m s}^{-2}$ (1)
 $50g = 490 \text{ m s}^{-2}$, and 864 m s^{-2} is larger, so the claim is correct (1)

B7 $\cos \omega t = 0.020/0.080 = 0.25$ (1) [3]
 $\omega t = \cos^{-1}(0.25) = 1.32 \text{ rad}$, with $\omega = 2\pi/1.2 = 5.24 \text{ rad s}^{-1}$ (1)
 $t = 1.32/5.24 = 0.25 \text{ s}$ (1)

SECTION C · Stretch

C1 (a) $\omega = 2\pi/T = 2\pi/0.50$ (1) [4]
 $\omega = 12.57 \text{ rad s}^{-1}$ (1)
 (b) $v_{\max} = \omega A = 1.01 \text{ m s}^{-1}$ (1)
 (c) $a_{\max} = \omega^2 A = 12.6 \text{ m s}^{-2}$ (1)

C2 $v = \frac{1}{2}v_{\max}$; $\frac{1}{2}\omega A = \omega\sqrt{(A^2 - x^2)}$ (1) [4]
 $\frac{1}{4}A^2 = A^2 - x^2$ (1)
 $x = (\sqrt{3}/2)A$ (1)
 $x = 0.087 \text{ m}$ (1)

C3 Velocity is a quarter of a cycle (90°) ahead of displacement (1), and is greatest [3]
 when displacement is zero (1). Acceleration is exactly out of phase (180°) with
 displacement, being greatest and opposite when displacement is at a maximum (1).

C4 a/x is the same for every pair: $-0.50/0.020 = -1.00/0.040 = -1.50/0.060 = -25 \text{ s}^{-2}$, [4]
 so $a \propto x$ (1)
 The acceleration is opposite in sign to the displacement, that is directed towards
 equilibrium, so the motion is SHM (1)
 Comparing with $a = -\omega^2 x$ gives $\omega^2 = 25 \text{ s}^{-2}$ (1)
 $\omega = 5.0 \text{ rad s}^{-1}$ (1)

C5 $A/2 = A \sin \omega t$ gives $\sin \omega t = \frac{1}{2}$ (1) [3]
 The first solution is $\omega t = \pi/6$ (1)
 With $\omega = 2\pi/T$, $t = (\pi/6)/(2\pi/T) = T/12$ (1)

C6 $a_{\max}/v_{\max} = \omega^2 A/(\omega A) = \omega$ (1) [4]
 $\omega = 14/0.90 = 15.6 \text{ rad s}^{-1}$ (1)
 $A = v_{\max}/\omega = 0.90/15.6 = 0.058 \text{ m}$ (1)
 $f = \omega/2\pi = 2.5 \text{ Hz}$ (1)