inkPhysics

ANSWER BOOK

The consequences of special relativity

Turning points · Year 13

AQA 3.12.3.3, 3.12.3.4, 3.12.3.5

18 questions · 53 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The time between the events measured in the inertial frame in which both events occur at the same place, the time on a clock that rides along with them (1). Every other inertial observer measures a longer time (1). [2]

A2 The length measured in the object's own rest frame (1). An observer the object passes measures it shorter along the direction of motion, and unchanged across it (1). [2]

A3 As v approaches c the mass $m_0/\sqrt{(1 - v^2/c^2)}$ grows without limit (1), so each further gain in speed costs ever more energy: reaching c would require infinite energy (1). [2]

A4 The energy equivalent of the particle's rest mass, $E_0 = m_0c^2$, measured in the frame in which it is at rest (1). [1]

A5 $E = m_0c^2$ (1) $= 1.67 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.5 \times 10^{-10}$ J (1). [2]

SECTION B · Standard

B1 $t = t_0/\sqrt{(1 - 0.95^2)}$ (1) [3]
$= 2.2/0.312$ (1)
$t = 7.0$ μs (1)

B2 $l = l_0\sqrt{(1 - 0.60^2)} = 120 \times 0.80$ (1) [3]
$l = 96$ m (1)
Contracted along the direction of motion only (1)

B3 Muons are created about 10 km up, moving near c, with a rest lifetime of 2.2 μs: one undilated lifetime covers only about 650 m, and the descent lasts around fifteen lifetimes, so almost none should survive (1). In fact a large fraction reach the ground (1). Their lifetime as measured from Earth is dilated roughly elevenfold (1), making the descent last barely one and a half lifetimes, and predicting the measured survival exactly (1). [4]

B4 $E = m_0c^2/\sqrt{(1 - 0.98^2)}$ (1) [3]
$E = 5.0 \times 8.2 \times 10^{-14}$ (1)
$E = 4.1 \times 10^{-13}$ J (1)

B5 $t = t_0/\sqrt{(1 - v^2/c^2)}$, so $\sqrt{(1 - v^2/c^2)} = t_0/t = 26/52 = 0.50$ (1). $v^2/c^2 = 1 - 0.25 = 0.75$, so $v = 0.87c$ (1) $= 2.6 \times 10^8$ m s^{-1} (1). [3]

B6 $l = l_0\sqrt{(1 - 0.996^2)}$ (1) = 10000 × 0.0894 = 894 m, about 890 m (1). The 10 km is the proper length, measured in the frame in which the atmosphere is at rest, the Earth frame; the muon measures the moving atmosphere contracted (1). [3]

B7 $E/E_0 = 1/\sqrt{(1 - v^2/c^2)} = 4.1 \times 10^{-13}/(8.2 \times 10^{-14}) = 5.0$ (1). So $1 - v^2/c^2 = 1/25$ (1), $v = 0.98c$ (1) $= 2.9 \times 10^8\ \text{m s}^{-1}$ (1). [4]

SECTION C · Stretch

C1 The root is $\sqrt{(1 - 0.80^2)} = 0.60$ (1) [4]
Time: 60 s is proper time, so Earth measures 60/0.60 (1)
t = 100 s (1)
Length: Earth measures 90 × 0.60 = 54 m (1)

C2 Electrons were accelerated through known pds, fixing their kinetic energy from eV, checked by the heat delivered to the target (1), and their speed was measured directly by timing their flight over a measured distance (1). As the energy rose, the measured speed flattened out just below c instead of climbing without limit (1): direct evidence that kinetic energy varies with speed relativistically and that c is a limiting speed (1). [4]

C3 The curve starts at the rest mass and stays almost flat to a few tenths of c (1), then rises ever more steeply, approaching a vertical asymptote at v = c that it never reaches (1). [2]

C4 In the Earth frame the journey takes 4.2/0.70 = 6.0 years (1). The crew's ageing is the proper time: departure and arrival both happen at the spacecraft (1). $\sqrt{(1 - 0.70^2)} = 0.71$ (1), so the crew age 6.0 × 0.71 = 4.3 years (1). 4.3 < 4.5, so the plan is met (1). [5]

C5 Classically $v = \sqrt{(2(e/m)V)} = \sqrt{(2 \times 1.76 \times 10^{11} \times 4.5 \times 10^6)} = 1.3 \times 10^9\ \text{m s}^{-1}$ (1). This is 4.2 times the speed of light, which nothing can reach (1). Bertozzi's time-of-flight measurements showed the speed instead levelling off just below c while the kinetic energy kept rising as supplied (1). [3]

C6 At low speeds the two curves coincide: the classical formula is a good approximation (1). From a few tenths of c the relativistic curve rises above the classical one, the kinetic energy growing without limit (1). It has a vertical asymptote at v = c that it approaches but never reaches, whereas the classical curve passes through c with only finite energy (1). [3]