InkPhysics

ANSWER BOOK

The Doppler effect and Hubble's law

Astrophysics · Year 13

AQA 3.9.3.1, 3.9.3.2 · CIE 7.3.1, 7.3.2, 25.3.1, 25.3.2, 25.3.3, 25.3.4
OCR A 5.5.3(d), 5.5.3(e), 5.5.3(f), 5.5.3(g), 5.5.3(h), 5.5.3(i), 5.5.3(j), 5.5.3(k), 5.5.3(l), 5.5.3(m), 5.5.3(n), 5.5.3(o)

19 questions · 58 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Every line in the spectrum arrives at a longer wavelength than the same line measured in the laboratory, the whole pattern stretched by one factor (1). The galaxy is receding, with $z = \Delta\lambda/\lambda$ giving $v = zc$ for speeds well below c (1). [2]

A2 The recession speed of a galaxy is proportional to its distance: $v = Hd$ (1). H is the gradient, about 65 km s^{-1} Mpc^{-1}: every megaparsec of distance adds 65 km/s of recession speed (1). [2]

A3 The cosmic microwave background, a 2.7 K black-body glow from every direction, the cooled afterglow of the hot early universe (1); and the roughly three-to-one hydrogen-helium mass ratio, matching fusion in the universe's first minutes (1). [2]

A4 $\Delta f/f = v/c$ in magnitude (equivalently $z = \Delta\lambda/\lambda$) (1); it holds only for speeds much less than the speed of light (1). Taking Δf as the observed frequency minus the rest frequency, a receding source gives a negative Δf and a positive $\Delta\lambda$, so $\Delta f/f = -v/c$ and $\Delta\lambda/\lambda = +v/c$ for recession speed v. [2]

A5 The star is approaching: motion towards the observer squeezes the wavelengths (a blue shift) (1). The fractional shift gives the line-of-sight speed, $v = c\Delta\lambda/\lambda$ (1). [2]

A6 $z = \Delta\lambda/\lambda$ compares the observed wavelength with the unshifted one, so the laboratory value is the baseline for the shift (1). [1]

SECTION B · Standard

B1 [3]

$z = (490.0 - 486.1)/486.1$ (1)

$z = 8.0 \times 10^{-3}$ (1)

$v = zc = 2.4 \times 10^{6}$ m s^{-1}, receding (1)

B2 [3]

$v = c\,\Delta\lambda/\lambda = 3.0 \times 10^{8} \times 0.080/550$ (1)

$v = 4.4 \times 10^{4}$ m s^{-1} (1)

The period of the wavelength oscillation is the orbital period of the binary (1)

B3 [2]

$d = v/H = 8800/65$ (1)

$d = 135$ Mpc (1)

B4 The Doppler shift responds only to the velocity component along the line of sight (1). Edge-on, each star's full orbital speed alternately points towards and away from Earth, giving the largest, cleanest wavelength swing (1); tilt the orbit and the measured shifts shrink (1). [3]

B5 G1's light is blue-shifted: it is approaching, so Hubble's law does not apply to it. G2, red-shifted, is receding (1). $z = (492.0 - 486.1)/486.1 = 0.0121$ (1), so $v = zc = 3.6 \times 10^6$ m s^{-1} = 3641 km s^{-1} (1). $d = v/H = 3641/65 \approx 56$ Mpc (1). [4]

B6 The size of the shift is $|\Delta f| = 1.420 - 1.402 = 0.018$ GHz (1). $v = c|\Delta f|/f = 3.0 \times 10^8 \times 0.018/1.420$ (1) $= 3.8 \times 10^6$ m s^{-1}, receding, since the received frequency is the lower (1). With Δf defined as observed minus rest, the shift itself is −0.018 GHz, and that negative sign is what recession means. [3]

B7 $v = Hd = 65 \times 220 = 14300$ km s^{-1} $= 1.43 \times 10^7$ m s^{-1} (1). $z = v/c = 0.048$ (1). $\Delta\lambda = 0.048 \times 656.3 = 31$ nm (1), so the line arrives at $656.3 + 31 \approx 688$ nm (1). [4]

SECTION C · Stretch

C1 [5]

$t = d/v = 1/H$ for every galaxy (1)

In SI units $H = 65\,000/(3.08 \times 10^{22})$ (1)

$H = 2.1 \times 10^{-18}$ s^{-1} (1)

$t = 1/H = 4.7 \times 10^{17}$ s (1)

Dividing by 3.156×10^7 s per year gives 1.5×10^{10} years, about 15 billion years (1)

C2 Space itself expands, carrying the galaxies apart: there is no centre and no edge (1), and an observer in any galaxy sees the same Hubble law (1). The student's picture cannot explain why the recession looks the same in every direction from here, and would from anywhere, nor why the microwave background arrives uniformly from the whole sky (1). [3]

C3 [4]

$v = zc = 0.30 \times 3.0 \times 10^8$ (1)

$v = 9.0 \times 10^7$ m s^{-1} (1)

$d = 90\,000/65 = 1385$ Mpc (1)

At nearly a third of light speed the small-shift approximation $z = v/c$ is strained, so both figures are estimates (1)

C4 The early universe was hot, dense and filled with radiation (1). The expansion of space has stretched (red-shifted) that radiation, cooling it (1); it now has a black-body spectrum at 2.7 K, peaking near one millimetre, as the model predicts for the cooled afterglow (1). It arrives almost uniformly from every direction, as radiation filling all of space should, rather than coming from any single source (1). [4]

C5 Each star alternately approaches and recedes along the line of sight as it orbits (1). Its line is blue-shifted while it approaches and red-shifted while it recedes, so the wavelength oscillates about the laboratory value (1). The two stars always move oppositely, and since each contributes a detectable copy of the line, it splits into two components that separate and merge as the stars orbit; were only one star bright enough to register, a single oscillating line would be seen instead (1). The period of the oscillation equals the orbital period of the binary (1). The maximum shift gives the orbital speed through $v = c\Delta\lambda/\lambda$ (1). Because the orbit is edge-on, the full orbital speed lies along the line of sight; a tilted orbit would give smaller shifts (1). [6]

C6 $H = v/d$: for A, $4550/70 = 65\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ (1); for B, $9750/150 = 65\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ (1). Both give the same constant, so speed is proportional to distance, consistent with Hubble's law with $H = 65\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ (1). [3]