



ANSWER BOOK

The time constant and exponential decay

Capacitance · Year 13

AQA 3.7.4.4 · CIE 19.3.1, 19.3.2, 19.3.3 · OCR A 6.1.3(b), 6.1.3(c), 6.1.3(d), 6.1.3(e)

18 questions · 49 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $\tau = RC$ (1); the time for the charge (or pd, or current) to fall to $1/e$ (about 37%) of its initial value (1). [2]
- A2** $\tau = RC = 10000 \times 100 \times 10^{-6}$ (1) [2]
 $\tau = 1.0 \text{ s}$ (1)
- A3** $Q = Q_0 e^{-t/RC}$, where Q_0 is the initial charge (1). [1]
- A4** 63% (the fraction $1 - 1/e$) (1). [1]
- A5** $\tau = RC = 2.2 \times 10^6 \times 47 \times 10^{-6}$ (1) [2]
 $\tau = 103 \text{ s}$ (1)

SECTION B · Standard

- B1** $\tau = RC = 2200 \times 470 \times 10^{-6}$ (1) [3]
 $\tau = 1.03 \text{ s}$ (1)
 After one time constant, $Q/Q_0 = 1/e \approx 0.37$ (37%) (1).
- B2** $\tau = RC = 47000 \times 100 \times 10^{-6} = 4.7 \text{ s}$ (1) [3]
 $V = V_0 e^{-t/RC} = 12 \times e^{-5.0/4.7}$ (1)
 $V = 4.14 \text{ V}$ (1)
- B3** $T_{1/2} = 0.69RC = 0.69 \times 4.7$ (1) [2]
 $T_{1/2} = 3.24 \text{ s}$ (1)
- B4** $R = \tau/C = 2.0/(220 \times 10^{-6})$ (1) [2]
 $R = 9091 \Omega$ (9.1 k Ω) (1)
- B5** $V = V_0 e^{-t/\tau}$, so $t/\tau = \ln(V_0/V)$ (1) [3]
 $\ln(6.0/2.2) = 1.00$ (1)
 $\tau = 15/1.00 = 15.0 \text{ s} \approx 15 \text{ s}$ (1)
- B6** $T_{1/2} = 0.69RC$ (1) [3]
 $C = T_{1/2}/(0.69R) = 4.6/(0.69 \times 33000)$ (1)
 $C = 2.0 \times 10^{-4} \text{ F}$ (202 μF) (1)

SECTION C · Stretch

- C1** $5.0 = 20e^{-t/3.0}$, so $e^{-t/3.0} = 0.25$ (1) [3]
 $t = -3.0 \times \ln(0.25)$ (1)
 $t = 4.16 \text{ s}$ (1)
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- C2** $\tau = RC = 10000 \times 470 \times 10^{-6} = 4.7 \text{ s}$ (1) [3]
 $V = V_0(1 - e^{-t/RC}) = 9.0 \times (1 - e^{-10/4.7})$ (1)
 $V = 7.93 \text{ V}$ (1)
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- C3** Taking logs of $V = V_0e^{-t/RC}$ gives $\ln V = \ln V_0 - t/RC$ (1); this is a straight line with [3]
gradient $-1/RC$ (1); so the time constant RC is the negative reciprocal of the gradient (1).
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- C4** The gradient is $-1/RC$, so $RC = 1/0.185 = 5.4 \text{ s}$ (1) [4]
 $C = RC/R = 5.4/10000 = 5.41 \times 10^{-4} \text{ F} = 541 \mu\text{F}$ (1)
The tolerance allows $376 \mu\text{F}$ to $564 \mu\text{F}$ (1)
 $541 \mu\text{F}$ lies inside this range, so the capacitor is within its stated tolerance (1).
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- C5** $E = \frac{1}{2}Q^2/C$, so E is proportional to Q^2 (1) [3]
After one time constant Q has fallen to Q_0/e , so E falls to E_0/e^2 (1)
 $1/e^2 = 0.135$, which is about 14% (1)
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- C6** $Q/Q_0 = 0.010 = e^{-t/\tau}$ (1) [3]
 $t = -4.0 \times \ln(0.010) = 4.0 \times \ln(100)$ (1)
 $t = 18.4 \text{ s}$ (about 4.6 time constants) (1)
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- C7** Charge the capacitor from a supply, then discharge it through the known [6]
resistor with a voltmeter or datalogger connected across the capacitor (or resistor) (1).
Record the pd at regular time intervals, using a stopwatch (or the datalogger's clock)
started as the discharge begins (1). Plot $\ln V$ against t (1); a straight line confirms that
the decay is exponential (1). The gradient of the line equals $-1/RC$ (1); so $C = -1/(R \times$
gradient), using the known resistance (1).
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